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Paper	Published	Discussion closes
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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

AXIOMS IN ROADWAY SOIL MECHANICS

By Henry C. Porter,[1] M. Am. Soc. C. E.

Synopsis

In the proper design and construction of roadways, the detailed study of soil mechanics in all of its phases is complicated by the many items that must be understood and taken into consideration. Many different types of soils must be investigated; their characteristics must be predetermined; and they must be properly used in the roadway. The mechanics of the different types of soils as placed in the roadway are different. Practical methods of designing and constructing different parts of the roadway are not the same. Topographical and climatic conditions in different localities vary and also must be taken into account. There is no panacea, and rules of thumb cannot be established which will fit all conditions. Those who design, construct, and maintain the highways must know the fundamentals of soil mechanics, and use their knowledge to fit each individual situation properly. On the other hand, there are many axiomatic facts, thirteen of which are presented in this paper for detailed examination and comment. These axioms pertain mostly to volumetric changes in soils due to fluctuations in moisture content caused directly by alternate seasons of slow rains and droughts.

Definitions

During the beginning of the study of soil mechanics, one's first impression is that there are so many factors involved that they never can be correlated sufficiently for practical usage. This is an erroneous impression. In the first place, topics which are not fully comprehended appear difficult and unreliable, but when understood thoroughly and put into usage, they are simple, and other phases of the subject then come to light from time to time. In the second place, many laws of nature involving soil mechanics which have been known by highway engineers for many years have been disregarded in the building of roadway "fills and embankments."

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **June 15, 1940.**

[1] Engr., Soils and Research, State Highway Dept., Austin, Tex.

Because different principles and methods are involved in the proper design and construction of various parts of the roadway, and in order that these parts may be referred to, conveniently and definitely, the roadway is divided vertically into three general parts, as illustrated in Fig. 1, and defined as follows:

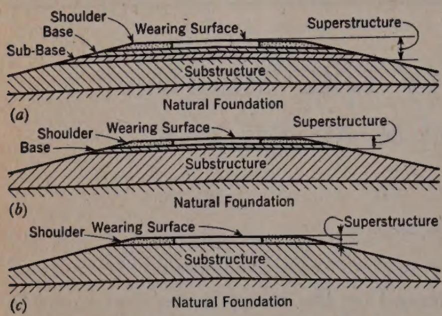

FIG. 1.—VERTICAL SUBDIVISION OF THE ROADWAY

A. The Natural Foundation: The non-manipulated natural structure upon which the "substructure," the subbase, the base, or the wearing surface, placed directly thereon, depend for their support;

B. The Substructure: The man-made part of the roadway lying between the top of the natural foundation and the bottom of the "superstructure," where the soil types therein are predetermined and the structure is designed in accordance with known mechanics and chemistry of the soils (fill and embankment are terms used where the soils were not predetermined and the structure was not designed in accordance with present knowledge of soil mechanics and chemistry, as described under the heading "Substructure"); and

C. The Superstructure: (1) The subbase, base, and wearing surface, or (2) the base and wearing surface, or (3) the wearing surface only, where no subbase or base are used (when the superstructure is built by stages, the first wearing surface eventually may become the subbase, or the base of the completed superstructure).

AXIOMS

Certain phases of soil mechanics and their effects on overlying riding surfaces are now so well known that they may be called axioms in roadway building. These axioms should be used immediately and constantly in the preparation of all natural foundations and in the design and construction of all substructures until their usage becomes a habit. Other findings concerning this subject then will be made from time to time and put into usage in such a way that the cost of construction and maintenance of the roadways will be reduced continually and the efficiency increased. These fundamentals, or axioms, where properly applied, will not only return the greatest and quickest dividends in roadway construction and maintenance, but also will make a good foundation upon which a practical knowledge of soil mechanics may be built.

Some of the most important known factors of soil mechanics involved in the proper design, construction, and maintenance of roadways are named and discussed as follows:

> Axiom 1. To a large extent, the behavior of the superstructure depends upon the nature and amounts of movements in the natural foundation and in the substructure.

Fig. 2 shows the smooth riding surfaces retained by the pavement where the soil substructure is non-expansive sand (profiles Nos. 1 and 2), and rough riding surfaces that developed shortly after completion of pavement on the same project, where the soil substructure is expansive clay soil (profiles Nos. 3 and 4).

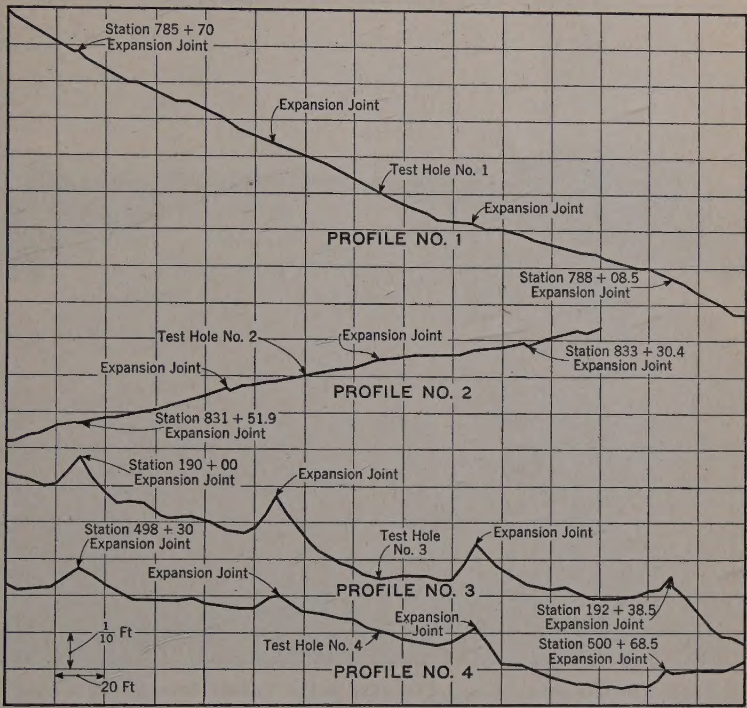

FIG. 2.—PROFILE AT CENTER LINE OF CONCRETE PAVEMENT

The pavement rose at the expansion joints where water leaked and wet the underlying clay soil.

Axiom 2. Many cohesive soils will support ordinary traffic loads without moving appreciably when they are relatively dry and are compacted in the natural foundation and substructure.

Axiom 3. Generally, the factor that causes density and volumetric changes or movements in the natural foundation and in the substructure is the flunctuation of moisture or water content in the soils.

A section of concrete highway, in Navarro County, Texas,[2] 1,500 ft long, was constructed in the winter and spring of 1931. It had a clay soil foundation of high volumetric change and was laid immediately after a long period of slow rains, when the soil was probably expanded to its maximum. Observations during the following summer indicated that there were more up and down movements along the edges where the moisture-content fluctuations were

[2] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Cambridge, Mass., Vol. II, 1936, p. 256; also Fig. 1, p. 257.

greater than along the center of the pavement. The fluctuations along the edges were 0.4 ft, whereas the movements along the center line were only 0.2 ft. The clay soil under the edges was exposed to seasonal wetting and drying more than the soil under the center of the pavement.

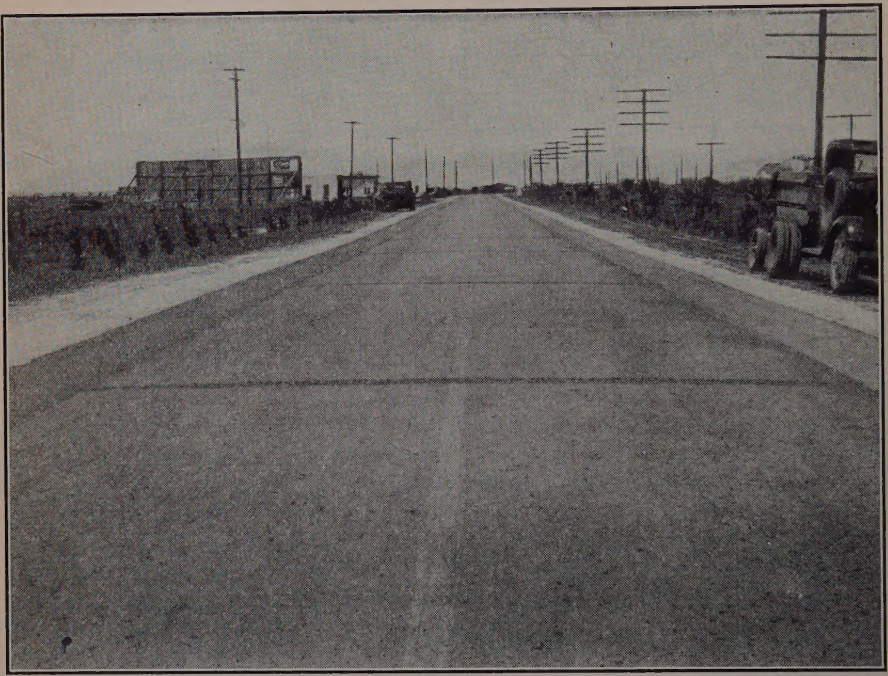

Fig. 3.—The Smooth Surface of a Pavement After Twenty-Four Years of Service

Axiom 4. Soils of different types behave differently when subjected to moisture-content fluctuations. Coarse-grained material of inappreciable volumetric change and plasticity, with moisture-content fluctuations, such as sand, does not move appreciably, whereas fine-grained, highly expansive and plastic clays do move appreciably.

This axiom is illustrated by Figs. 3 and 4, showing the riding surface of a pavement after 24 years of service. This pavement carries heavy seaport traffic and, for more than a week after the 1919 tidal wave along the Texas seacoast, it was submerged beneath several feet of water. This highway has retained its smooth riding surface continuously with small maintenance costs.

To study the reasons for this long life, a section of the pavement was cut away. It was found that the underlying soil was composed of sand and shells which do not move with moisture-content fluctuations. At the place shown in Fig. 4, the pavement was only 2.5 in. thick.

Axiom 5. A sloping smooth surface of compacted, fine-grained, clay soil tends to shed rain water, but when water is retained in contact with clay soil, continuously, the water eventually will permeate the clay sufficiently to cause it to expand in volume and become plastic.

Axiom 6. When rain water first falls on coarse-grained, pervious soil, such as sand, the line of least resistance to the movement of the water is into the large spaces or voids in the sand. Where sand rests on a sloping clay soil, the water will percolate through the sand until it arrives at the top of the impervious clay and the tendency then is for the water to move laterally through the sand along the top of the clay.

Water is furnished to most wells by movement of the water through strata of sand and gravel—sometimes through seams and crevices in shale and rock, but never in appreciable quantities through fine-grained clay soil. Generally, clay is used for making jugs, and sand is used for filters. Where the sand layer rests on and is flanked by clay soil on all sides, rain water often is impounded in the large voids of the sand on the clay soil and eventually permeates the clay soil sufficiently to cause it to expand in volume and become plastic.

Four views of a model test to study rainfall saturation are shown in Fig. 5. In Fig. 5(*a*) the rain is simulated by the manipulation of a flask and a number of wicks. It is shown running along the surface of the sloping clay soil. In this instance the sand (which extends to the intersections of the side slopes) does

Fig. 4.—Section of Pavement in Fig. 3 Uncovered to Study Reasons for Its Long Life

not impound the water on the under-lying clay soil; and, consequently, the clay soil has not been wet appreciably in 45 min. The reverse effect is shown in Fig. 5(*b*) in which the sand was entrenched in the clay soil. In that case the water had wet the underlying clay appreciably in 45 min.

After testing the foregoing two models for 19.5 hr, the results were as shown in Figs. 5(*c*) and 5(*d*). In that time, Fig. 5(*a*) became saturated to the extent shown in Fig. 5(*c*). In this case shrinkage cracks in the clay soil permitted the water to penetrate deeply into the soil substructure at those places.

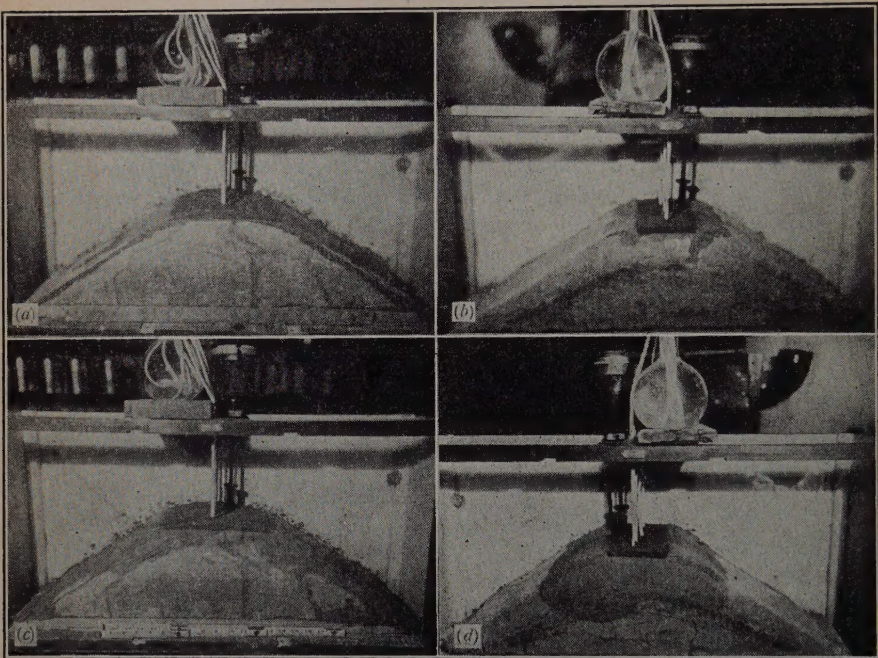

FIG. 5.—MODEL STUDIES OF RAINFALL SATURATION

Fig. 5(*b*) after 19.5 hr of simulated rainfall is shown in Fig. 5(*d*). All the water has been absorbed by the clay soil. No water has run down the side slopes of the embankment, and the clay soil at the toes is still dry.

In 1935, observations were made on a section of Texas highway that had been laid on a 12-in. layer of sand in a trenched clay subgrade.[3] It was found that, after long periods of slow rain, water rose through contraction joints to the top of the pavement and trickled along the top.

Holes were dug at the edge of the pavement in two adjacent sections. In Fig. 6(*a*) the shoulder was clay, and as soon as the hole was dug the water began to flow from the sand beneath the pavement. After one hour water had accumulated in the hole to a depth of 3.5 in.

At the adjacent section, conditions were the same as in Fig. 6(*a*) except that the 12-in. layer of sand extended as a blanket entirely across the crown to intersections with the side slopes of the roadway. Water has never risen to the top of the pavement at this section, and the test hole (see Fig. 6(*b*)) remained entirely free of water.

[3] "Roadbed Design for Self-Drainage," by Henry C. Porter, *Engineering News-Record*, June 16, 1938, Fig. 1.

The centrifuge moisture equivalent (CME) test gives information as to the relative porosities or draining qualifications of different types of soils, and is one of the most important of the soil-constant tests. The determination of the relative draining characteristics of the different soils used is necessary for the proper design, construction, and maintenance of modern roadways, not only for the superstructure, but also for the substructure and the natural foundation.

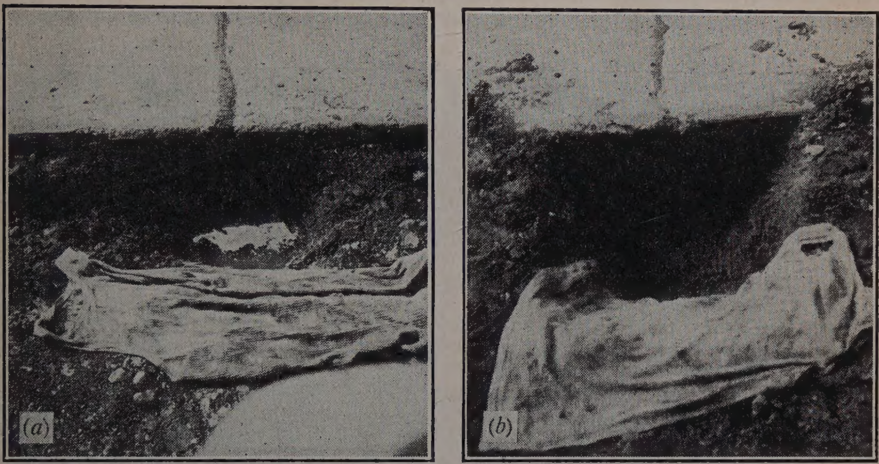

FIG. 6.—COMPARATIVE TEST SECTIONS AT EDGE OF PAVEMENT

> Axiom 7. When the moisture content of expansive, fine-grained, clay soil increases above the shrinkage limit of the soil, the soil expands in volume, and the expanding soil will exert great lifting force.

It is known that some expanding clays will raise a weight of 50 lb per sq in. appreciably.

> Axiom 8. When the soil type beneath the superstructure is uniform, when the soil is compacted to a uniform density with a uniform moisture content at the time the superstructure is laid, and when the subsequent moisture-content fluctuations in the soil are uniform throughout the entire length and width of the superstructure, then the superstructure will move up and down uniformly and will retain its smooth riding surface.
>
> Axiom 9. The conditions enumerated in Axiom 8 can be obtained only by proper preliminary investigations, design, supervision of construction, and maintenance.

Often, merely treating the so-called "subgrade" is not sufficient to overcome the subsequent ill effects of all mechanical defects in fills and embankments and in natural foundations. Adequate preliminary investigations, design, and supervision of construction are necessary primarily because of the following phenomena:

(*a*) Much of the natural structure was formed by sedimentation and is in layers of materials of different types—perhaps a layer of clay, a layer of caliche, and a layer of sand, gravel, or rock. For example, in Fig. 7, the top layer is

sandy loam, the second is clay, the third is sand, the fourth is clay, and the fifth stratum is gravel with a calcareous binder. Hence, when a pavement is laid to a grade on natural undisturbed structure (on an eroded hill-slope or in a hill-cut, for example), the foundation of the pavement may be composed of materials that will move by different amounts when subjected to moisture-content fluctuations. The changes in the amounts of movements very likely will be abrupt both in the foundations and in the overlying pavement. This is usually the cause of the development of uncomfortable riding surface irregularities in

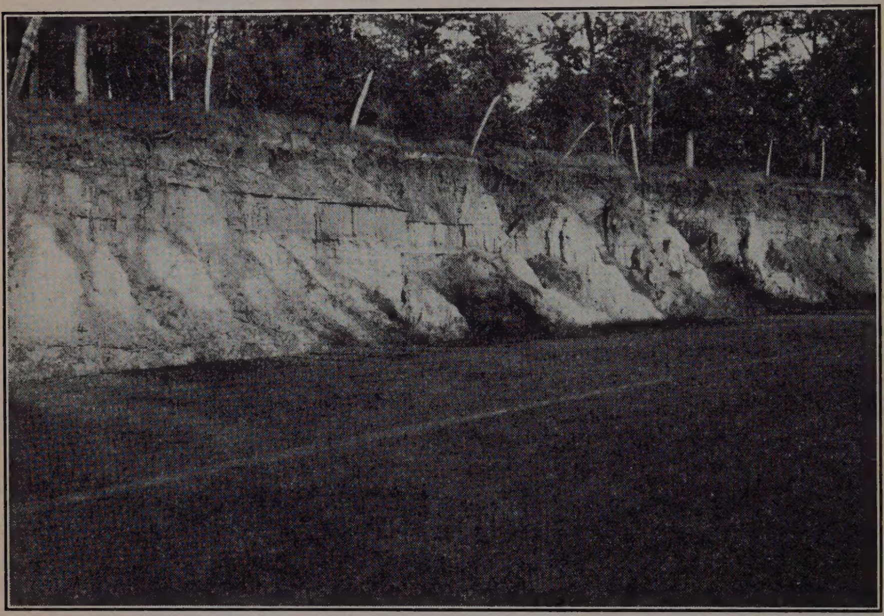

Fig. 7.—Stratified Natural Formations of Materials of Different Types

cuts, on hill-slopes, and at the junction of cuts and fills. During construction, irregular natural soil formations in a cut should be excavated to below bottom-of-superstructure grade line and backfilled in such a manner that the undisturbed natural structure will not cause the development of irregularities in the superstructure.

(*b*) Materials of heterogeneously deposited natural formations often must be used for constructing substructures. If these materials are placed in fills and embankments (without selection), as generally has been done in the past, the change from highly expansive to non-expansive soil in the fills will be abrupt in many instances. Furthermore, pockets of coarse-grained soil often will be formed which, eventually, will impound water in the fills, as illustrated in Figs. 5 and 6. To overcome these mechanical defects the nature of the soils must be predetermined, by means of the prepared soil profile, and the substructure properly designed—even as superstructures are designed now. Where soils of different types must be used, the most impervious and highly expansive

soil generally should be placed at the bottom of the substructure where the soil will be protected most from rain water moisture-content fluctuations, and so on, until the most pervious and non-expansive soil is placed at the top of the substructure. The top of each layer of soil should slope continuously from the center line to intersections with the roadway side slopes, for internal drainage. Where a layer of coarse-grained, pervious material is placed on impervious soil, particular care must be taken that the coarse-grained material is not flanked by finer-grained and less pervious soil.[4] When it becomes necessary to make a longitudinal change from one type of soil to another in any particular layer, the change should be gradual so that there will be no subsequent abrupt longitudinal difference in movements.

> Axiom 10. Because of extreme wet and dry seasons in a large part of the United States, it is impracticable at present to prevent all moisture-content fluctuations in soil foundations and in soil substructures; but it is practicable to control the fluctuations to such an extent that their ill effects on the superstructure will be reduced, if not eliminated.

In some localities, all of the soil is uniform in type, but is highly expansive, and plastic, clay. Such material must be used as the natural foundation and for building the substructure. In these cases, provisions must be made for maintaining uniform moisture-content fluctuations, as nearly as is practicable, throughout the entire natural foundation and substructure after the pavement is laid. Where the pavement leaks, such as at expansion joints and contraction cracks in concrete pavement, and at cracks and breaks in bituminous topped pavements, the underlying clay soil becomes wet, expands, and therefore raises the pavement more at these concentrated places than elsewhere. As a result, objectionable riding surface irregularities develop.

The development of objectionable riding surface irregularities will be reduced greatly (particularly at points where changes in soil types unavoidably occur) if the upward movements from the original elevations of the pavement which occur during wet seasons are equal to the downward movements from the original elevations of the pavement which occur during droughts. If the upward movements are equal to the downward movements, the variation from the original elevations or smooth riding surface of the pavement will be a minimum. To accomplish this objective, the moisture content, density, and volume of the soil at the time the pavement is laid on it must be the mean of that which subsequently will prevail, as nearly as practicable. If clay soil is compacted to its maximum density with its optimum moisture at the time the pavement is laid, subsequent wetting of the soil during rainy weather will cause the pavement to rise above its original elevations. It will never subside to below its original elevations, and the movement will be a maximum from the original elevations.

If the soil is saturated with water and expanded to its maximum volume when pavement is laid, the pavement will never rise above its original elevations. When the excess moisture is lost during dry weather, the pavement will subside and the amount of movement from the original will be a maximum.

[4] "Roadbed Design for Self-Drainage," by Henry C. Porter, *Engineering News-Record*, June 16, 1938, Fig. 5.

Suppose one half the length of a highly expansive soil embankment is compacted to its maximum density and the other half is saturated with water (expanded to its maximum volume) at the time the pavement is laid on each half. If extended dry weather follows completion of the work, the first half will not move, the second half will subside, and an irregularity in the riding surface of this pavement will develop at the junction of the two halves. On the other hand, if the completion of the pavement is followed by slow rains which appreciably raise the moisture content in the soil of the first half, this pavement will rise above its original elevations, the pavement on the second half will not move, and an irregularity in the riding surface will develop at the junction of the two halves.

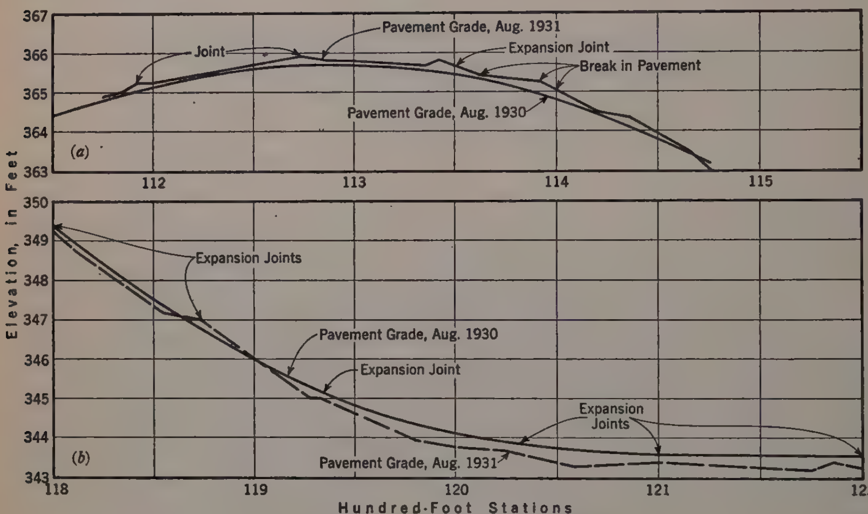

FIG. 8.—CHANGES IN PAVEMENT GRADE

This phenomenon is illustrated in Fig. 8. The lower line of Fig. 8(*a*) shows the profile of the top of a concrete pavement when it was finished in August, 1930. The soil was relatively dry and compacted at the time the pavement was laid, the surface being wet to a depth of 10 to 12 in. There was a great quantity of rain during the following fall and winter, at which time surface irregularities developed. By August, 1931, all this section of pavement had risen above its original elevations—more so at the expansion joints than elsewhere. The upper solid line of Fig. 8(*b*) shows the profile of the top of the pavement on another section of the same project (adjacent to it), when it was finished in August, 1930. The embankment soil on this section was jetted and ponded with water three weeks prior to the laying of the pavement. By August, 1931, this section of pavement had subsided from its original elevations, but not as much at the expansion joints (which perhaps had leaked) as elsewhere.

An expansion joint, without dowels, was the junction of the two projects.[5]

[5] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Cambridge, Mass., Vol. II, 1936, p. 258; also Figs. 5 and 7.

The natural soil on the two projects was of the same type of highly expansive clay. One pavement was laid during a long period of intermittent slow rains about the first of 1931. At the time this section of pavement was laid, the soil fill was saturated with rain water and perhaps expanded to its maximum volume. Later in 1931, after considerable dry weather, the pavement was laid on the second project when the soil was relatively dry and in a shrunken condition. During the drought of 1934 the pavement on the first project subsided as much as 2.5 in. more than did the pavement on the second project, which caused an irregularity to develop in the riding surface at the junction of the two projects. During the following winter and spring of 1934–1935, there were many slow rains. By June, 1935, the slab-ends lacked only $\frac{5}{8}$ in. maximum of being back to the same elevations.

Where there is non-uniformity of moisture content and density in the clay soils at the time pavement is laid thereon, riding surface irregularities develop both during extreme wet seasons and during droughts.

The foregoing comments have been concerned with compacted, coarse-grained material, such as sand, which does not change in volume with moisture-content fluctuations, and with fine-grained clays which expand in volume when their moisture contents are increased above their shrinkage limits. There is another axiom which also must be taken into consideration—the bulking of sand when it contains a small quantity of moisture and is disturbed or manipulated.

> Axiom 11. When water is added to undisturbed sand until the sand is saturated or inundated, the water tends to compact or decrease the volume of the sand; but when damp sand is manipulated, it bulks.

It has been demonstrated repeatedly that when a quantity of dry sand is stirred while water is added slowly to it, the sand rapidly bulks as more and more water is added, until, with a certain quantity of moisture, the sand reaches a maximum volume. The increase in volume of the sand, when measured in the bulked condition, is much greater than the volume of the water added. The mixing of 5% or 6% of water by weight may cause the sand to increase in volume as much as 20% or even 30%. Further additions of water tend to recompact the sand and decrease its volume as more and more water is added. When sand is saturated, or inundated, the volume of the sand is approximately the same as when it is measured dry. The degree of bulking varies with the type of sand. If the dry sand is not stirred or disturbed while the water is poured on it, the volume of the sand will not change appreciably.

This characteristic of sand should be borne in mind when pavement is placed on a sand fill or blanket. If sand is not perfectly dry or is not entirely saturated with water at the time pavement is placed on it, it is likely to be in a bulked condition. If the sand is bulked, eventually it will very likely become saturated with water under the pavement, settle, and cause the pavement to subside, or leave it suspended on an air pocket or pockets to carry traffic loads by the beam strength of the pavement.

This phenomenon is illustrated by observations made on a project where the plans called for a 12-in. layer of sand blanket to be placed entirely across the crown of clay fills, with concrete pavement to be laid on the sand. For the

convenience of hauling materials on the clay shoulders, the sand first was placed in a trench cut in the clay soil to the 20-ft pavement width; the pavement was laid on the sand and was cured by ponding water on it. After the ten-day curing period, when the clay shoulder soil was cut away to the bottom of the sand under the pavement in order to extend the sand blanket to intersections with the roadway side slopes, air pockets were found between the top of the sand and the bottom of the pavement. The sand was wet after being placed in the trench; but afterward it probably was disturbed during fine-grading and shaping operations, and consequently bulked. After the pavement was laid, curing water probably reached the bulked sand and caused it to subside from the bottom of the pavement. If the sand had been wet thoroughly after it was fine-graded and shaped, these settlements in the sand would have developed and been remedied before the pavement was laid.

The foregoing comments have dealt to a large extent with the proper longitudinal design of the soil substructure. The transverse design also is important, particularly in the widening of old pavements. The phenomena are practically the same.

> Axiom 12. The mechanics of the soils must be given proper consideration in the transverse as well as the longitudinal design and construction of the roadway.

Before plans are drawn for pavement widening, the soil structure underlying the old pavement should be investigated thoroughly, its condition determined, and the plans drawn accordingly. For illustration of the importance of the preliminary investigation: In 1937, plans were drawn (without preliminary soil

FIG. 9.—EMBANKMENT SETTLEMENT OF 4 IN. UNDER THE OLD CONCRETE SLAB WHEN WATER IS PONDED ON THE SHOULDER SOIL TO PREPARE FOR WIDENING THE PAVEMENT

embankment investigations) for widening an old concrete pavement 11 years after the soil embankment was built and 7 years after the 10 ft width was built. When water was ponded on the old shoulder soil in preparing to widen the pavement to the 20 ft width in 1937, non-uniform settlements occurred in the

soil, as shown in Figs. 9 and 10. These settlements occurred not only in the soil of the shoulders where the new pavement was to be placed, but also under the edge of the old pavement.

If the new width of pavement had been laid on the shoulder soil without ponding water on it first, non-uniform settlements eventually would have occurred and caused cracks and riding surface irregularities to develop in both the new and old pavement. This embankment was sandy soil and was in a bulked condition when the original concrete pavement was laid thereon in 1930.

Fig. 10.—Ill Effects of Foundation Settlement

If the preliminary investigations show the underlying fill and natural formations beneath the old pavement to be structually sound, the type of soil, moisture content, and density of the soil substructure for the extended width of pavement should be made as nearly like the old part as is practical. An illustration of the subsequent ill effects of a difference in type of construction in the two parts of fills and embankments are illustrated by observations and findings made in 1937. In this instance the old 9-ft width of concrete placed in 1924 is on clay soil. In 1933, when the pavement was widened to the 20-ft width, 12 in. of sand were placed on the old clay shoulder soil and the 11 ft additional width of concrete was placed on the sand. The shoulders of the new concrete were built of clay soil. Subsequent movements in the abutting different types of construction have been different and caused a 2-in. maximum crack to form between the old and new pavement in some places. In this case the major movement differential was transverse instead of vertical. This is one type of riding surface failure caused by lack of uniformity in designing the two parts of the roadway.

During the hot dry weather, the clay shoulder soil on the first of the two projects, previously mentioned[5] subsided as much as 5 in. below the top edge of the pavement. It has been found that when these shoulders are built up with

the same type of clay soil during the dry weather, the shoulder soil re-expands when the rainy season comes until it extends several inches above the top of the pavement and must be cut away. If concrete for widening is laid on this shoulder soil during dry weather, when the soil is in a shrunken condition, the soil eventually will become wet, expand in volume, and raise the new pavement above the old.

> Axiom 13. In order to build roadways with pavements that will not develop appreciable riding surface irregularities, one or two major items must be embodied in the construction plans:
>
> (*a*) The natural soil foundation and soil substructure must be of materials which will not move appreciably with moisture content fluctuations; or,
>
> (*b*) The moisture-content fluctuations in expansive and plastic soils must be controlled to such a degree that subsequent movements will not be appreciable.

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PAPERS

MEASURING THE POTENTIAL TRAFFIC OF A PROPOSED VEHICULAR CROSSING

By N. Cherniack,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The appraisal of the economic practicability of proposed self-liquidating toll crossings, financed by debenture bonds which are secured only by a lien on, and which must be serviced by, revenues from tolls, has raised technical problems of traffic and revenue estimating which are much more difficult than those encountered in the economic justification of free and tax-supported highway facilities.

This paper analyzes the potential traffic of any proposed vehicular facility into a number of elements, indicates and discusses the factors which, past experience has shown, determine the sizes of those elements, and suggests methods of estimating the elements by adequate measurements of their determinant factors. It also indicates the difficulties encountered in testing the suggested methods by checking estimated against realized traffic, after the opening of new facilities for which traffic estimates had been prepared.

Introduction

Need for Modern Vehicular Crossings.—It is common knowledge that in the two decades since the World War there has been a phenomenal growth in the use of motor vehicles for private and commercial transportation of people and freight, and that the rapidly mounting volumes of vehicular traffic have induced an expansion in highway construction. However, the much heavier expenditures required for the construction of adequate modern bridges and tunnels than for equal stretches of tributary highways has caused the building of such facilities to lag considerably behind highways. This lag, ultimately, has given rise to urgent and vociferous demands for modern river crossings.

Problem of Financing Crossings.—In the decade 1920–1930, cities and states could afford to satisfy those demands with tax-supported, toll-free bridges,

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 15, 1940.

[1] Statistical Analyst, The Port of New York Authority, New York, N. Y.

because they could usually borrow the necessary funds on the basis of their general credit. Consequently, in that decade of rising taxes and expanding tax yields, elaborate studies of the economic justification of highway facilities were almost superfluous. However, when tax yields began to decline, when taxpayers began to demand relief, and when governors and mayors began to find it increasingly difficult to balance state and municipal budgets, they were no longer willing (and in many cases they were unable) to borrow the necessary funds to construct even urgently needed highway bridges or vehicular tunnels which were exceedingly costly in comparison to similar stretches of highways. How to meet demands for these facilities, in the face of equally urgent state and municipal needs, created real financial problems for these officials.

Where the demand for a vehicular crossing had grown to such proportions that it became evident that to meet it would constitute a profitable venture, private companies were organized to build and operate toll crossings, usually under state franchise. A number of such toll bridges were so constructed and have been financially successful, whereas others, for various reasons, have not.

Public Financing.—At the same time, in the period under discussion, a relatively new governmental idea was being tested. It consisted of setting up a governmental agency and charging it with the duties of planning, financing, and operating such public facilities for which there was a definite and large enough demand to make the project self-sustaining in character. This agency, at first supported by its governmental parent, was to investigate the economic practicability of such facilities, and, if satisfied that they would be self-liquidating in character, was to proceed to finance such a facility on the merit of the specific project and on the sole credit of such agency.

Such a governmental agency is The Port of New York Authority. It was created by compact between the states of New York and New Jersey, which compact was ratified by Congress in April, 1921. It has planned, financed, and constructed, and now (1940) has in operation four interstate bridges connecting New York and New Jersey. It purchased from the states of New York and New Jersey, and operates, the Holland Tunnel under the Hudson River. In the midst of the economic depression, it financed the Lincoln Tunnel under the Hudson River, between West 38th Street, Manhattan, and Weehawken, N. J., at first through the Federal Works Administration, and then subsequently refinanced the loan through private investment bankers. The first tube of this tunnel was opened to traffic on December 22, 1937.

It is extremely doubtful whether the State of New Jersey, for example, could have afforded to contribute its share of the cost of constructing these four bridges and the new Lincoln Tunnel, and then to have operated them as free facilities, covering the fixed and operating charges through general taxation. More likely, these crossings would not have been built. On the other hand, this bi-state agency, by 1939, will have spent $200,000,000 for the construction of its bridges and tunnels, which have tremendously expedited the movement of vehicles, people, and freight between the two states and within the New York metropolitan area. At the same time, neither for the construction of these facilities nor for their operation, has it cost, nor will it cost, the New Jersey taxpayer (or, for that matter, the New York taxpayer), a single tax dollar.

It is the motorist who has been willing to pay for the entire cost of both construction and operation of these crossings on an "as-and-when-you-desire-to-use-them" basis. This is evidenced from the fact that the motorist patronage which these crossings have been receiving has made them, as a group, financially self-sustaining right through the most serious and unprecedented economic depression in history.

Thus it has been demonstrated definitely that, rather than be deprived of urgently needed, faster, more convenient, and safer bridges and tunnels, because they could not be built as free, tax-supported crossings, the motoring public has preferred to have such facilities built and operated as toll crossings and has given them strong financial support with their loyal patronage.

Toll Versus Free Crossings.—The appraisal of the economic practicability of self-liquidating toll crossings for financing purposes, however, has raised problems exceedingly more difficult than that of the economic and social justification of free, tax-supported highway facilities, and for the following reasons:

(1) A free facility is supported on a broad tax base; a toll facility on a narrow revenue traffic base;

(2) Taxes are mandatory, whereas tolls are optional;

(3) The success of a free facility is measured by its relief to congestion; that of a toll crossing by the traffic volume it attracts; and

(4) The economic justification of a free facility may be predicated on liberal valuations of benefits, because traffic estimates need never be checked. The economic practicability of the toll crossing must be sound because revenue estimates must meet the "cash-register test."

Elements of the Proposed Crossing's Traffic.—The staff of The Port of New York Authority, assisted by personnel of the U. S. Works Progress Administration (WPA), has made studies of vehicular toll crossings opened since 1927. Such studies have indicated that these new crossings have obtained their vehicular traffic from the following sources, from which any future proposed crossing might also derive its traffic:

(1) Existing traffic: Diversions of vehicular traffic from existing alternate routes between tributary areas; and

(2) New traffic: (*a*) "Normal" expansion of existing vehicular traffic; and (*b*) vehicular traffic "generated" by the new crossing.

Measuring Existing Traffic

Delimiting Its "Traffic-Shed."—To measure the existing vehicular traffic volume that could make use of the new crossing, it is necessary first to delimit "traffic-sheds" tributary to the proposed crossing, one on each side of the stream. The boundaries of passenger car and truck "traffic-sheds" may be ascertained from a time contour map constructed from data collected through a series of comprehensive automobile test runs (simulating both passenger cars and trucks) between centers of traffic density and via every alternative existing route, the travel time via proposed routes, of course, being estimated.

Measuring Its Aggregate Traffic "Reservoir."—Having delimited the vehicular "traffic-sheds," it is then necessary to determine the vehicular and passenger

traffic volumes which now flow between the sheds on either side of the stream. These volumes, constituting the aggregate potential traffic "reservoir," must be determined for annual periods, since the revenues to be predicated upon these potential traffic volumes must be calculated on an annual basis.

Annual vehicular traffic volumes may usually be compiled where toll crossings are in operation. However, where part or the entire traffic "reservoir" consists of traffic over free bridges or highways, records of annual traffic volumes are usually not available. In these cases, annual volumes must be estimated from data collected for short periods. Such estimates must be prepared carefully and the probable maximum errors determined and kept within practical limits.[2]

A too optimistic estimate of the annual volume in the traffic "reservoir" may become the first serious "weak spot" in the financial forecast of the proposed crossing. Thus, a large volume of traffic in the summer should not be taken as a positive indication of a correspondingly large annual volume; nor heavy Sunday traffic to indicate heavy weekday traffic; nor peak "rush-hour" traffic a high daily volume. Seasonal, daily, and hourly "patterns" of vehicular traffic flows for the routes which form the traffic "reservoir" of the proposed crossing should be developed carefully and, with their aid, estimates of annual traffic should be prepared on the basis of sufficiently adequate samples.

It will be seen from Table 1(*a*) that an estimate based on a 1-hr count in the peak morning or evening rush hour (during which about 7% of the 24-hr traffic usually moves) would be subject to a probable maximum error of more than 50%, and that it requires a 12-hr count to reduce this error to about 15%. However (see Table 1(*b*) and 1(*c*)), if the percentage moving in the 12-hr period is determined from a 24-hr count for a specific crossing (such as the Holland Tunnel or George Washington Bridge), the error of an estimate of the 24-hr traffic of that crossing in the future, based on a 12-hr count, may be reduced to less than 5%.

Table 2 contains some daily indexes of traffic based on annual 1938 traffic. They are expressed in percentages of the five-day average, Monday to Friday, which is considered as 100%. It will be seen that: (*a*) Wednesday's and Thursday's percentages are closest to a weekday average; and (*b*) Saturday and Sunday percentages (of an average weekday) vary widely. Traffic volumes taken on normal Wednesdays or Thursdays will approximate weekday traffic volumes closely, but they will not indicate the corresponding Saturday or Sunday volumes; nor could Saturday or Sunday volumes be used to indicate weekday volumes. Consequently, a midweek day and at least a Sunday volume must always be observed to arrive at an estimate of the complete weekly traffic volume.

Table 3 shows monthly adjusted median indexes expressed in percentages of an average month (eliminating statistically the effects of growth and reflecting only seasonality) for the following types of vehicular crossing:

(*A*) 2 bridges connecting recreational with residential areas (Marine Parkway Bridge, New York City; and Buffalo Peace Bridge, Buffalo, N. Y.).

[2] "Methods of Estimating Vehicular Traffic Volume with the Aid of Traffic Patterns," by N. Cherniack, *Proceedings*, Highway Research Board, November, 1936.

TABLE 1.—MEDIAN PERCENTAGES AND MAXIMUM PERCENTAGE DEVIATIONS FROM MEDIANS (FROM 24-HR TRAFFIC VOLUMES)

Size of Sample: No. of hours	Hours From:	Hours To:	(a) 21 Selected Facilities[a]: Median percentage of 24-hr volume	(a) Maximum deviation from median percentages	(a) Maximum percentage deviation from median percentages	(b) Holland Tunnel: Median percentage of 24-hr volume	(b) Maximum deviation from median percentages	(b) Maximum percentage deviation from median percentages	(c) George Washington Bridge: Median percentage of 24-hr volume	(c) Maximum deviation from median percentages	(c) Maximum percentage deviation from median percentages
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Weekday											
12	7 a.m.	7 p.m.	70.9	8.5	12.0	71.4	2.0	2.8	69.9	1.9	2.7
6	1 p.m.	7 p.m.	36.9	5.5	14.9	37.6	1.6	4.3	37.7	1.3	3.5
6	7 a.m.	1 p.m.	33.9	6.1	18.0	33.9	2.1	6.2	31.9	1.7	5.3
3	4 p.m.	7 p.m.	21.2	6.7	31.6	21.8	1.0	4.6	33.8	1.2	5.0
3	7 a.m.	10 a.m.	17.4	7.7	44.3	16.5	1.2	7.3	17.5	1.5	8.6
1	5 p.m.	6 p.m.	7.2	3.8	52.8	....	....	....	....	....	....
1	8 a.m.	9 a.m.	7.0	3.6	51.4	....	....	....	....	....	....
Saturday											
12	7 a.m.	7 p.m.	67.4	5.2	7.7	68.6	1.6	2.3	70.5	1.3	1.8
6	1 p.m.	7 p.m.	36.0	8.7	24.2	35.8	1.2	3.4	41.4	1.1	2.7
6	7 a.m.	1 p.m.	29.5	11.1	37.6	32.4	0.7	2.2	29.4	1.0	3.4
3	4 p.m.	7 p.m.	17.5	7.2	41.1	17.2	0.7	4.1	20.8	1.5	7.2
3	7 a.m.	10 a.m.	13.5	7.0	51.9	14.4	0.5	3.5	13.9	0.7	5.0
Sunday											
12	8 a.m.	8 p.m.	64.1	10.6	16.5	59.3	1.2	2.0	65.3	1.9	2.9
6	2 p.m.	8 p.m.	39.3	9.4	23.9	32.3	0.8	2.5	37.5	2.0	5.3
6	8 a.m.	2 p.m.	26.0	4.3	16.5	27.4	1.1	4.0	27.8	1.3	3.5
3	5 p.m.	8 p.m.	19.9	7.1	35.7	17.8	0.4	2.2	21.5	1.2	5.6
3	8 a.m.	11 a.m.	9.5	5.0	52.6	10.8	1.0	9.3	10.7	0.9	8.4

[a] List of crossings: Holland Tunnel, George Washington Bridge, Bayonne Bridge, Goethals Bridge, Outerbridge Crossing, Brooklyn Bridge, Manhattan Bridge, Williamsburgh Bridge, Queensborough Bridge, Victory Bridge, Pulaski Skyway, George A. Posey Tube, Sumner Tunnel, Madison Avenue Bridge, Willis Avenue Bridge, Washington Bridge, Third Avenue Bridge, McCombs Dam Bridge, University Heights Bridge, Ship Canal Bridge, 145th Street Bridge.

TABLE 2.—DAILY INDEXES OF VEHICULAR TRAFFIC FOR SELECTED PORT AUTHORITY CROSSINGS AND THREE TYPICAL HUDSON RIVER FERRIES

Day	George Washington Bridge	Holland Tunnel	Lincoln Tunnel	Bayonne Bridge	Goethals Bridge	Outerbridge Crossing	125th Street Ferry	Electric Ferries	Cortlandt Street Ferry
Monday	103.6	102.9	98.2	102.4	99.3	101.8	102.0	103.1	103.9
Tuesday	94.8	94.5	94.4	96.0	96.9	94.7	97.4	96.6	97.3
Wednesday	95.1	96.8	99.0	100.0	99.2	96.4	98.7	98.3	96.9
Thursday	94.8	97.4	99.4	99.4	99.4	98.7	99.4	99.7	99.2
Friday	111.7	108.4	109.0	102.3	105.3	108.5	102.5	102.4	102.9
Five-day average	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Saturday	137.3	113.3	102.8	102.2	107.8	140.5	119.5	92.8	70.1
Sunday (or holiday)	191.8	131.4	106.4	150.2	145.4	204.7	146.2	94.7	34.4

(*B*) 5 bridges connecting suburban residential areas (Ambassador Bridge, Detroit, Mich.; Rip Van Winkle Bridge, Catskill, N. Y.; Mt. Hope Bridge, Bristol, R. I.; Tacony-Palmyra Bridge, Palmyra, N. J.; and Outerbridge Crossing, Perth Amboy, N. J.).

(*C*) 9 bridges and 3 tunnels connecting urban residential with commercial areas (Mid-Hudson Bridge, Poughkeepsie, N. Y.; George Washington Bridge, Fort Lee, N. J.; Bayonne Bridge, Bayonne, N. J.; Goethals Bridge, Elizabeth, N. J.; Golden Gate Bridge, San Francisco, Calif.; Philadelphia-Camden Bridge, Philadelphia, Pa.; Carquinez Bridge, Vallejo, Calif.; Triborough Bridge, New York City; Henry Hudson Bridge, New York City; Sumner Tunnel, Boston, Mass.; Mersey Tube, Liverpool, England; and Antwerp Tunnel, Antwerp, Belgium).

(*D*) 2 tunnels and 1 bridge connecting commercial with industrial areas (Holland Tunnel, Jersey City, N. J.; Posey Tube, Oakland, Calif.; and San Francisco-Oakland Bay Bridge, San Francisco, Calif.).

TABLE 3.—MEDIAN MONTHLY INDEXES[a] OF VEHICULAR TRAFFIC FOR FOUR TYPES[b] OF VEHICULAR TOLL CROSSINGS

Month	No. of days in month	Group (*A*)	Group (*B*)	Group (*C*)	Group (*D*)
January	31	38.6	53.3	73.1	92.6
February	28	31.7	52.5	67.4	84.4
March	31	44.5	66.8	82.0	94.6
April	30	54.9	78.5	92.9	100.1
May	31	89.7	107.7	110.9	105.9
June	30	138.4	120.8	113.8	104.3
July	31	260.0	172.9	133.8	108.9
August	31	254.3	181.9	132.9	108.6
September	30	126.0	131.6	114.1	101.6
October	31	68.4	93.8	102.4	102.0
November	30	48.2	76.0	90.7	98.6
December	31	45.3	64.2	86.0	98.4
Average	30.4	100.0	100.0	100.0	100.0
Year	365	1,200.0	1,200.0	1,200.0	1,200.0

[a] Expressed as percentages of an average month considered as 100.0, reflecting varying number of days in each month and seasonality, but growth of traffic eliminated statistically. [b] See definition of groups in the text.

Each of the four sets of indexes represents adjusted medians selected, in turn, from the median indexes (medians of monthly indexes for a series of years) of 20 individual crossings in the four groups.

It will be seen that the spring months of April and May and the fall months of September and October come closest to an average month for the year. Hence, traffic-volume observations should be made in those months, if practicable. If observations are made in the summer or winter months, care must be used to place the facility under observation, at least, into one of the four types presented herein.

Measuring Its Share of the Existing Traffic "Reservoir."—A decade or so ago, the notion was prevalent that the new toll bridges or tunnels would completely "dry up" competitive ferries, and would "inherit" all of their traffic when they ceased operating. Experience, since then, has shown that ferries continue to operate despite the competition of toll bridges and tunnels.

On the other hand, doubt has been expressed as to whether a toll bridge could divert sufficient traffic from free bridges to compete successfully with them. That doubt has been dispelled by the operation of the Triborough and Henry Hudson bridges in New York, N. Y., which have diverted substantial traffic volumes from the nearby free Queensborough and Ship Canal bridges, over the East and Harlem rivers, respectively.

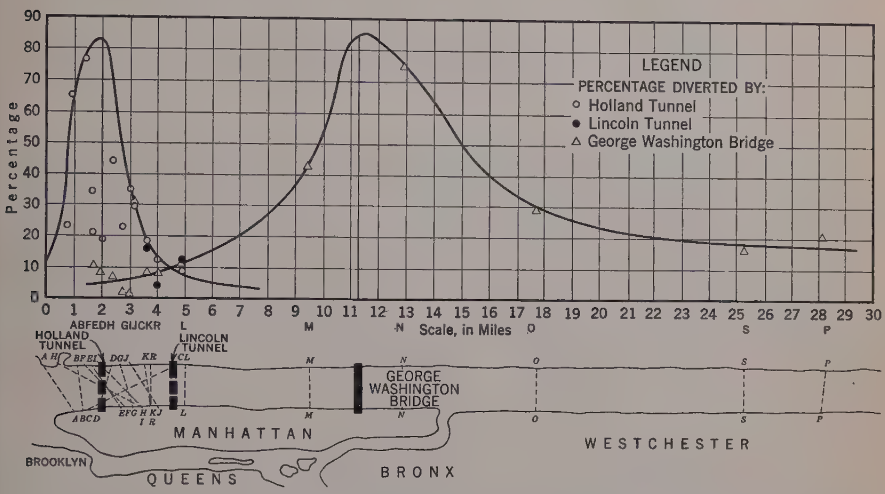

Fig. 1.—Percentages of Competitive Trans-Hudson Vehicular Traffic Diverted by the Holland Tunnel and George Washington Bridge at the End of the First Year of Their Operation

Consequently, the problem of measuring, accurately, the share of the existing traffic "reservoir" which a proposed toll crossing can divert from competitive facilities or alternative free routes is still difficult, and requires careful study to avoid the second possible serious weak spot in the estimates.

TABLE 4.—Diversion Experience Factors of Selected Toll Crossings

Crossings	Opening date of new crossing	Competitive Ferry Lines[a]		Traffic diverted, in percentages[e]
		Number	Name	
Holland Tunnel	November 13, 1927	13	Hudson River ferries[b]	46.9
Goethals Bridge	June 29, 1928	1	Elizabeth Ferry	47.6
Outerbridge Crossing	June 29, 1928	1	Perth Amboy Ferry	65.9
George Washington Bridge	October 25, 1931	5	Hudson River ferries[c]	49.1
Bayonne Bridge	November 15, 1931	1	Bayonne Ferry	23.7
Triborough Bridge	July 11, 1936	2	East River ferries[d]	12.7
Oakland Bay Bridge Golden Gate Bridge	November 12, 1936 May 27, 1937	....	Ferries	61.5
Lincoln Tunnel	December 22, 1937	3	Nearby ferries	11.6

[a] Ferry routes from which bridges and tunnels diverted substantial amounts of traffic. [b] 2.5 miles or less from the Holland Tunnel. [c] 18 miles north, and 2.5 miles south, of the bridge. [d] Also Queensborough Bridge (free), 3.2 miles from the bridge. [e] Percentage of traffic volume of competitive ferries in a 12-month period preceding the opening of the new crossing, and diverted by it in its first year of operation.

Experience factors of diversions by toll crossings which have been opened are available, but such factors show a wide variation. In general, however, they indicate that the new crossing has diverted the largest percentages of

traffic: (*a*) From crossings in its proximity, and (*b*) from those crossings with the smallest favorable toll spreads (see Table 4 and Fig. 1). In Fig. 1, the letters along the base line denote approximate locations of ferry lines.

Measuring Diversions (Origin and Destination Basis).—For the preparation of accurate estimates of the traffic divertible from alternate routes, the following information is essential:

(*a*) Annual traffic volumes of every alternate route, segregated by "lines of travel," connecting pairs of small areas, one on either side of the river;

(*b*) Travel times and mileages between the centers of areas of traffic density;

(*c*) Tolls via all alternate crossings; and

(*d*) Physical and psychological characteristics of all alternate routes, such as type of highway, scenery, slum areas, direct or "zigzag," traffic lights, congestion, and other characteristics influencing the choice or avoidance of given routes.

Segregating the Traffic of the Crossing by "Lines of Travel."—Traffic volumes of alternate routes are rarely available, segregated geographically. Consequently, segregations by "lines of travel" must be determined from sample

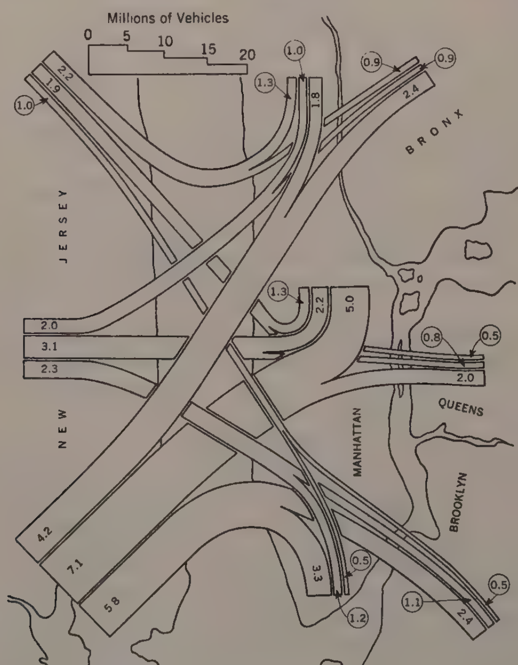

FIG. 2.—NINE TRANS-HUDSON "LINES OF TRAVEL"

origin and destination counts, compiled in sample traffic volumes between pairs of zones on either side of the river (see Fig. 2). The zones should be small enough to reveal significant differences in travel time, mileage, and other route characteristics.

Having thus segregated the annual traffic volumes of individual crossings by "lines of travel," similar "lines" between the same pairs of zones, but via alternate routes, may then be combined for all routes to determine the total travel between pairs of small areas (see Table 5). It is then necessary to determine the portions of each "line of travel," now traveling via all existing routes, that will be diverted by the proposed crossing.

TABLE 5.—ANNUAL 1935 TRANS-HUDSON VEHICULAR TRAFFIC FOR ONE OF NINE MAJOR "LINES OF TRAVEL" SEGREGATED INTO FOUR OF 56 "LINES OF TRAVEL"

Crossing	(a) Teaneck, N. J., and Lower Manhattan		(b) Teaneck, N. J., and Brooklyn, N. Y.		(c) Hackensack, N. J., and Lower Manhattan		(d) Hackensack, N. J., and Brooklyn, N. Y.	
	Vehicles	Percentages	Vehicles	Percentages	Vehicles	Percentages	Vehicles	Percentages
George Washington Bridge	39,657	12.7	43,683	21.4	46,192	19.4	111,555	39.7
125th Street Ferry	39,130	12.6	42,832	21.0	23,682	10.0	31,766	11.3
42d Street Ferry	31,523	10.1	19,799	9.7	23,253	9.8	20,384	7.3
Electric Ferries	90,286	29.0	49,453	24.2	39,487	16.6	37,717	13.4
Holland Tunnel	61,490	19.7	29,038	14.2	70,313	29.6	52,879	18.8
Remaining 12 crossings	49,408	15.9	19,613	9.5	34,634	14.6	26,661	7.5
Total	311,494	100.0	204,416	100.0	237,751	100.0	280,862	100.0

Factors Governing Choice of Alternate Routes.—Experience has shown that the new crossing is not likely to divert all the traffic between any pair of zones (even if it were the fastest or shortest route), but would share it with the alternate routes in proportion to its relative advantages.

Usually, there is no one obviously "best" route among them which is at once the shortest, quickest, cheapest, and most convenient. Frequently, the shortest route is likely to be congested, and hence not the quickest. The quickest route may often be longer and may run via a toll bridge or tunnel, with a higher toll than on competing crossings. The most direct route, between a given origin and destination, may involve steep grades and poorly paved highways and approaches. Certain other characteristics, such as directness or "zigzag," scenery or slums, parkways or congested narrow streets, and others, also influence the choice or avoidance of a route. Under such conditions motorists are compelled to weigh time, distance, tolls, convenience, directness, and other factors in selecting what each considers the "best" route at any given time.

Therefore, it is essential for the investigator to travel over all alternate routes between centers of traffic density on either side of the river, and determine travel times, mileages, tolls, and those other physical and psychological characteristics at different times. It is also desirable to consult residents of the areas for information as to unusual local conditions that might influence the choice of routes at different times of the day and in different seasons of the year.

Then again, a salesman, for example, bound for a "prospect," might place such a premium on time as to disregard a higher toll entirely; but on his return trip he might choose the cheapest, and perhaps the longest route. A truck driver might choose the shortest and cheapest route, at the expense of valuable truck time. A pleasure-bound motorist might, at times when he is "flush," disregard tolls and choose the quickest and most scenic route, whereas at other times he may choose the cheapest crossing. Thus an individual motorist may choose any route at one time and another route at another time between the same points of origin and destination. It is to be expected, therefore, that motorists in the aggregate, traveling in the course of a year between any area of origin on one side of the river and any area of destination on the other, could be found to have distributed their patronage between a number of alternative crossings. Studies have corroborated this observation and have shown that, somehow, motorists in the aggregate appear to "rate" alternative routes on the basis of their relative attractiveness along individual "lines of travel" by distributing their mass patronage as they do.

The "relative merit rating" of alternate routes for any given "line of travel" may be appraised by expressing in dollars and cents not only differences in tolls from those of a standard route, but also differences in trip time, mileage, and other physical and psychological route characteristics, and thus arriving at a relative single trip-cost factor (see Eq. 7 in the Appendix). By computing the patronages obtained by alternate routes relative to standard routes along a large number of "lines of travel" and plotting these relative patronage factors against relative "cost" factors (considering the measurable factors of tolls, running time, waiting time, distance, and their monetary evaluations), it was found that the two factors (relative "cost" and relative patronage) were related. This relationship could best be expressed by Eqs. 6 and 7 in the Appendix. The "scatter" of the data points on such charts indicated, however, that other factors (not directly or as readily measurable as are the factors of distance, running time, waiting time, and tolls) also affected the choice or avoidance of any individual route, under given circumstances, and hence its "relative merit rating." Such other factors may be safety, travel habit, avoidance of industrial or slum areas, attractiveness of scenic parkways, etc. Their effects were determined, as a group, after allowance had been made first for the effects of the measurable factors. They may be referred to as "preference" or "prejudice" factors and are represented in Eq. 7 as another cost factor (ΔC_p) which diminishes or augments the combined toll, travel time, and distance cost differences.

In Eq. 6, whereas the cost factor (ΔC) reflects the quality of the crossing, the discount factor (d) reflects the keenness of competition from its competitors. The greater the number of competitor crossings, the closer together they are, and the easier it is (by reason of the street and highway layouts) to switch

from an original intention of using one crossing to that of using an alternate without undue "backtracking," the keener is the competition and hence the greater the discount factor (d).

It is probably true that very few motorists actually compute, in dollars and cents, the value of each trip between traffic centers and via different crossings. However, they probably do have a sense of value which governs their choice of possible routes between their origins and destinations on different occasions, and on the basis of this sense of value motorists in the aggregate apparently evaluate running and waiting time and distances in some proportion to each other and to the tolls, the evaluations of which may be determined in any particular study by means of origin and destination data, auto test runs, and data on tolls and other route characteristics.

By reason of these relationships (Eqs. 6 and 7), it is possible to establish an approximate measure of the "relative merit rating" of any proposed crossing which would also reflect its probable relative patronage provided its "cost" factor (ΔC) (in Eqs. 6 and 7) and the discount factor (d in Eqs. 6) could be determined from observed data.

Estimating Divertible Traffic Shared by the New Crossing.—The "relative merit ratings" of existing crossings may be determined in two ways: (*a*) From the manner in which any given "line of travel" is now distributed among them, and (*b*) from their route characteristics. The "relative merit rating" of the proposed crossing, on the other hand, must be determined by comparing its probable route characteristics with those of existing routes. By determining the "relative merit rating" of the proposed crossing (either by judgment or by the use of Eqs. 6 and 7 in the Appendix), it is possible to estimate its probable share of any given "line of travel" by computing the ratio of its "relative merit rating" to the sum of the ratings of all its competitors (see Eq. 5 in the Appendix).

For example, Col. 3, Table 6, shows the annual volume of trans-Hudson vehicular traffic moving along the "line of travel" between Passaic, N. J., Clifton, N. J., and vicinity, on the west side of the Hudson River, and the Manhattan area, 72d Street to 125th Street, east of the Hudson River, distributed among several individual crossings and the remaining crossings combined. The percentage shares handled by these crossings are shown in Col. 4 (also see Eq. 3 in the Appendix). Considering the crossing handling the largest share of this "line of travel" as having a rating of 100, the merit ratings of the individual alternate crossings are obtained by computing the ratios of their patronage to that of the best crossing. Col. 6 shows differences in travel cost, obtained by evaluating running-time differences shown in Col. 7 at 1 cent per min, waiting-time differences shown in Col. 8 at 2 cents per min, and distance differences shown in Col. 9 at 3 cents per mile, and then adding, algebraically, the toll differences shown in Col. 10 (also see Eq. 7 in the Appendix). In Col. 11 the ratings of existing crossings are shown transcribed from Col. 5. The rating of the proposed crossing shown as Item 9 in Col. 11 was obtained on the basis of judgment, using Eqs. 6*a* and 7 in the Appendix as a guide, and by comparing the cost differences of the proposed crossing with those of the existing crossings shown in Col. 6. In Col. 12 is shown the probable percentage share which each crossing (both existing and proposed) would have obtained had the

new crossing been in operation in 1935. These shares were computed by dividing the rating of each crossing by the sum of the ratings of all crossings (existing and proposed); see Eq. 5 in the Appendix. Col. 13 shows the probable redistribution of the existing traffic on the basis of the percentage shares shown in Col. 12. Thus, it will be seen that along this "line of travel" the proposed crossing would have obtained 15,481 vehicles, had it been opened in 1935.

TABLE 6.—Distribution of Annual 1935 Trans-Hudson Passenger-Car Traffic Along One "Line of Travel," and the Estimated Redistribution Had the Lincoln Tunnel Also Been in Operation in 1935

Item No.	Crossing	Symbol	Under Existing Conditions			Differences from "Popular" Crossing					Assuming Lincoln Tunnel in Operation[f]		
			Annual number of passenger cars (1935)	Percentage of total	Rating factor[d] (percentages)	Net cost, in cents[e]	Running time, in minutes	Waiting time, in minutes	Distance, in miles	Toll, in cents	Rating factor	Percentage of total	Number of vehicles
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	125th Street Ferry[a]	M	23,312	32.5	100[d]	....	41	7	11.8	0	100	25.5	18,276
2	George Washington Bridge	GWB	18,447	25.7	79	+16	− 5	−7	+3.3	+25	79	20.1	14,405
3	Holland Tunnel	HT	11,374	15.9	49	+48	+13	−7	+8.0	+25	49	12.5	8,959
4	West 42d Street[b]	L	9,031	12.6	39	+30	+15	+1	+1.6	+ 8	39	9.9	7,095
5	Electric Ferries	R	4,436	6.2	19	+30	+17	0	+2.8	+ 5	19	4.8	3,440
6	West 23d Street[c]	K	2,115	3.0	9	+37	+14	+5	+3.3	+ 3	9	2.3	1,648
7	Remaining crossings (7)	....	2,954	4.1	13	....	....	....	....	....	13	3.3	2,365
8	Existing crossings	....	71,669	100.0	308	....	....	....	....	....	308	78.4	56,188
9	Lincoln Tunnel	....	....	....	....	+10	−10	−7	+3.2	+25	85	21.6	15,481
	Total of Items 8 and 9	....	....	....	....	....	....	....	....	....	393	100.0	71,669

[a] For 125th Street Ferry—for this "line of travel," the "most popular" crossing—the actual time-distance and tolls are shown. [b] To West Shore Railroad, New York Central System. [c] To Delaware, Lackawanna and Western Railroad. [d] The rating factor of the crossing that handled the largest volume of traffic along an individual "line of travel" is considered as 100%. [e] Net cost, computed at 1 cent per min of travel time, 2 cents per min of waiting time, and 3 cents per mile of distance. [f] Redistributed annual 1935 passenger-car traffic.

By summarizing the portions of all "lines of travel" (into which the entire trans-Hudson traffic volume had been divided and which the proposed crossing would divert from all existing alternate routes), the total annual volume of divertible traffic in the base year may thus be obtained.

Implications of This Method of Estimating Diversions.—From the fact that the share of traffic of any "line of travel" which a crossing obtains is determined from the ratio of its rating to the sum of the ratings of all crossings (see Eq. 5 in the Appendix), it will be seen that any crossing's share thus depends upon the following factors: (*a*) The quality of the crossing itself, as reflected in its merit rating; (*b*) the number of its competitors, as reflected by the number of ratings by which its rating is divided to obtain its share; and (*c*) the respective qualities of its competitors, as reflected in the numerical values of their ratings.

Thus, the number and value of the merit ratings of individual competitive crossings determine the manner in which traffic along any individual "line of travel" is distributed among all crossings competing for each "line." For the same reason, traffic along different "lines of travel" is usually distributed differently among all crossings.

Changes in the factors that affect the ratings of any one or more of a group of competitive crossings will bring about a redistribution, from time to time, among the competitive crossings of the traffic along the "lines of travel" affected. Since crossing ratings are affected by changes in net costs via individual routes (see Eq. 7 in the Appendix), changes in ratings may be brought about by: (1) Revision in tolls (ΔT in Eq. 7); (2) changes in travel times ($\Delta T_r; \Delta T_w$ in Eq. 7) or distances (ΔD in Eq. 7) between origins and destinations, as a result of highway improvements; (3) changes in convenience, scenic attractiveness, slums, industrial areas, and other travel characteristics (ΔC_p); and (4) changes in economic conditions ($\Delta C_r; \Delta C_w; \Delta C_d; \Delta T$ in Eq. 7).

Thus, for example, changes in tolls on any one or more of a group of competitive crossings will change the differences in costs between those crossings and the standard crossing, and not the cost differences between the remaining crossings and the standard, thereby raising the ratings of the former and, consequently, increasing the shares of traffic of the former on all "lines of travel." Similarly, the construction of a new highway would reduce travel times via only one or more of a group of competitive crossings, thus reducing net cost differences from the standard on those crossings and not on others. The effect would be to increase the ratings of the one group of competitive crossings and not those of the others, and to increase, especially, the shares of those "lines of travel" that were affected by the new highway.

In times of business depression, furthermore, motorists' evaluation of differences in time, distance, and convenience is reduced considerably, whereas differences in tolls loom large, thus bringing about higher ratings, and hence greater patronage on the lower toll, but slower, less convenient crossings.

To summarize, then, the share of traffic along any given "line of travel," which any existing crossing obtains, or any new one would obtain, is determined at any given time by: (*a*) Its own quality, based upon the motorists' evaluation of its relative travel characteristics; (*b*) the aggregate number of its competing crossings; and (*c*) their qualities. The share of traffic is determined at different times by: (*d*) Changes in its own or its competitors' relative travel characteristics, such as toll, travel time, distance, and convenience differentials, from those of some standard crossing; (*e*) changes in motorists' evaluation of those travel characteristics; and (*f*) a widening of the choice of routes.

In actual practice, the difficulties of forecasting the share which a new crossing would divert from existing crossings lies in the changes in the different factors that are constantly taking place, the lack of data (which usually are costly to obtain) required to establish the changing travel characteristics of competitive routes, and the changing monetary values placed by motorists upon travel characteristics.

The major difficulties in checking estimates of divertible traffic from each "line of travel" are due to the fact that, after the opening of the new

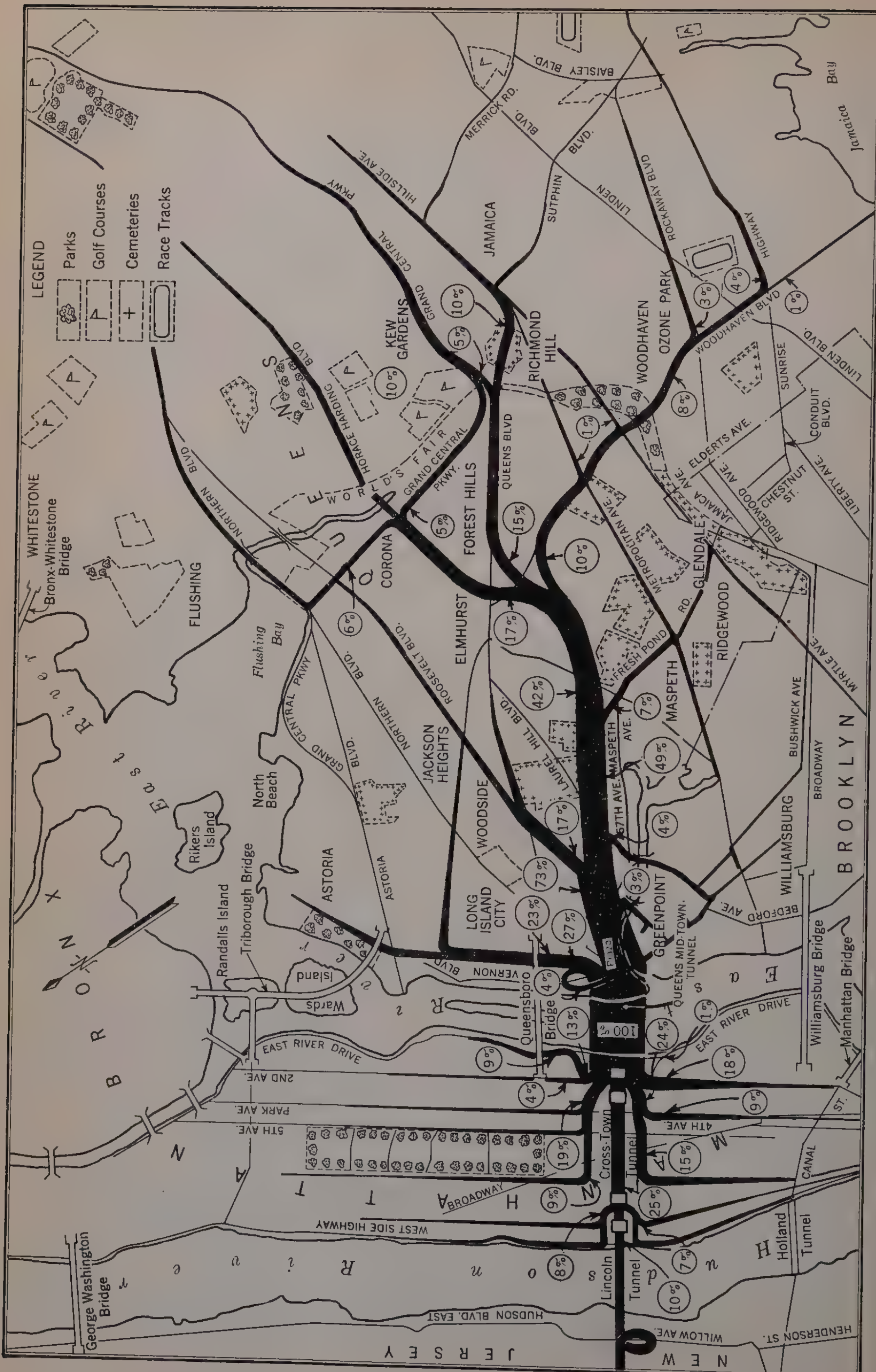

FIG. 3.—ESTIMATED DISTRIBUTION OF QUEENS MIDTOWN TUNNEL TRAFFIC ALONG EXISTING AND PROPOSED ARTERIAL ROUTES

crossing, there are usually no origin and destination data corresponding to those collected in connection with the preparation of estimates, by reason of the costliness of collecting such data. Moreover, contemporaneous with diversions, "generation" of traffic by the new facility takes place in varying degrees along the different "lines of travel." Other difficulties are discussed elsewhere in this paper.

By-Products of This Method of Estimating Diversions.—Attention is called to several important by-products that may be obtained as a result of estimating the divertible traffic of the proposed crossing by the foregoing method.

In the first place, the probable percentage share of the "reservoir" which the proposed crossing would divert can now be obtained readily by computing the ratio of its integrated, elemental, divertible volumes from individual "lines of travel" to the total in the traffic "reservoir" in the base year. This percentage may then be applied to the annual traffic "reservoir" compiled in any future year (assuming basic conditions remain approximately the same), and the divertible traffic for any other year may thus be determined.

In the second place, it is possible to compute the probable percentage diversions from each competitive crossing, and thus obtain information which is exceedingly helpful, subsequently, in comparing estimated diversions from each competitive crossing against the actual diversions that may be currently compiled after the crossing has been opened.

As a third by-product, it is possible to determine, for each side of the river, the probable geographical distribution (at least of the divertible portion) of the proposed crossing's traffic, from which, assuming that traffic prefers the fastest, most convenient, and most direct route between the proposed crossing and the origin or destination on each side of the river, it is possible to prepare a traffic flow map showing the probable routing of traffic along the streets and highways, as well as the approaches to the crossing on each side of the river (see Fig. 3).

The resulting flow lines of the potential traffic of the proposed crossing may then be superimposed upon the existing streets and highways and may be examined: (*a*) To determine which of the existing streets and highways will be called upon to handle the various portions of the proposed crossing's traffic; and (*b*) to ascertain their adequacy to do so, in the light of the local traffic which they now are handling and will probably handle in the future. Such an examination also will usually suggest the highways that are destined to be the "natural feeders" to the proposed crossing, and a field examination will indicate which highways will prove inadequate and which must be supplemented by improvements and new construction. Changes, improvements, and new construction can then be recommended that will produce the system of arterial streets and highways necessary to handle the traffic of the proposed crossing rapidly, expeditiously, safely, and conveniently, and thus make for the realization of its potential traffic as forecasted (see Fig. 4).

Measuring the New Traffic

Need for Measuring "Normal" Traffic Expansion.—Frequently it is discovered that the probable reasonable percentage share of the existing traffic

FIG. 4.—EXISTING AND RECOMMENDED ARTERIAL HIGHWAYS IN THE BOROUGH OF QUEENS SERVING THE QUEENS MIDTOWN TUNNEL

volume "reservoir" which the proposed crossing is likely to divert at a predetermined schedule of tolls would produce insufficient revenues to insure its economic practicability. In that case, either of two policies could be adopted:

(1) To await the time when the "visible" traffic "reservoir" actually will have expanded to a point where its percentage share will be large enough to make the proposed crossing self-liquidating at the outset; or

(2) To anticipate a probable expansion of the transportation needs of the community and plan the facility so that it can be completed at the earliest time when potential revenues are likely to render the project self-supporting over the life of the structure.

The first policy—that of waiting until its potential traffic is completely "visible"—is likely to discourage existing travel so much that the necessary volume may perhaps never be realized.

The other side of the picture is this: If traffic expansion were not really anticipated and the proposed facility were merely expected to handle part, or all, of the existing traffic (even if more adequately) there would scarcely be the incentive for financing a new modern facility to replace or supplement existing facilities.

The second policy, on the other hand (that of anticipating traffic expansion), compels the engineer, although he may be averse to it, to peer into the future. Therefore, he must develop scientific methods of estimating future traffic expansion that are sufficiently reliable to gain the confidence of financiers.

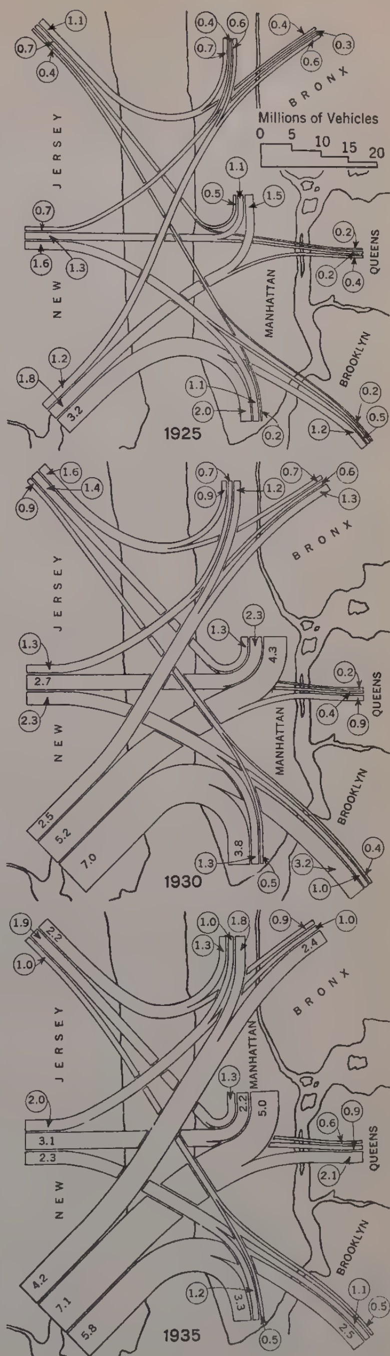

FIG. 5.—ANNUAL TRANS-HUDSON VEHICULAR TRAFFIC, DISTRIBUTED AMONG NINE MAJOR "LINES OF TRAVEL"

Empirical Projection of Traffic.—Any method of estimating future trends must rely to a greater or lesser extent on past experience, modified to embody the probable effects of known or estimated future conditions. It would also be desirable, of course, if the different traffic volumes originating in the several areas tributary to the proposed crossing could be obtained for a series of years. Such data, however, are usually not available. Fortunately, in connection with studies of the George Washington Bridge and Lincoln Tunnel, and with the aid of WPA personnel in recent years, origin and destination data have been collected and compiled in 1925, 1930, and 1935. These data permit one to obtain a glimpse of the radically different rates of expansion which occur in traffic volumes originating in different areas, some of which are tributary to the new facility, and some of which are not (see Fig. 5).

Where such data are not available, the nearest approach to these data which have been compiled in recent years is the total annual traffic volumes of alternate crossings, tapping areas of which only some are common to those tributary to the proposed crossing.

Prior to 1930, when economic depressions appeared to have had no effect on traffic expansion, annual rates of expansion could be derived from such annual traffic volume series, and were expressed in several ways, such as:

(*a*) Constant average annual traffic volume increments per year (straight line) (see Eq. 8 in the Appendix);

(*b*) Equivalent average percentage of the preceding year's volume (straight line on logarithmic paper or compound interest curve) (see Eq. 9 in the Appendix);

(*c*) Variable annual volume increments dependent on available margins of capacity ("Pearl Reed" curve) (see Eq. 10 in the Appendix); and

(*d*) Other more complicated mathematical equations.

Past rates, expressed empirically, could be used either unadjusted or adjusted. and applied to past volumes to project them into the future (see Fig. 6).

Yardsticks of Traffic Expansion.—The foregoing empirical projections related traffic growth to the mere passage of time and thus impelled the inevitable assumption of continual growth. They were thus manifestly defective. It was obviously more desirable to relate traffic growth to what might be termed "traffic determinants," which would permit anticipation of shrinkages as well as expansions in traffic volumes. Several traffic determinants readily suggest themselves: Population, motor vehicle registration, opening of new crossings, economic conditions, and other expansion and shrinkage factors.

Population.—Population tributary to the crossing is obviously a fundamental determinant of traffic. Nevertheless, as a factor in vehicular traffic expansion in the past, it has perhaps been least important, other factors having eclipsed its effects (see Fig. 7). Moreover, since population census data are available only once in every 10 years, estimates available during the intercensal years are too unreliable to be of value as quantitative aids in projecting traffic into the future. Even under a stationary population, vehicular traffic may still expand substantially under the effects of greater purchasing power available for adult recreational activities, of which motor travel is one of the most universal.

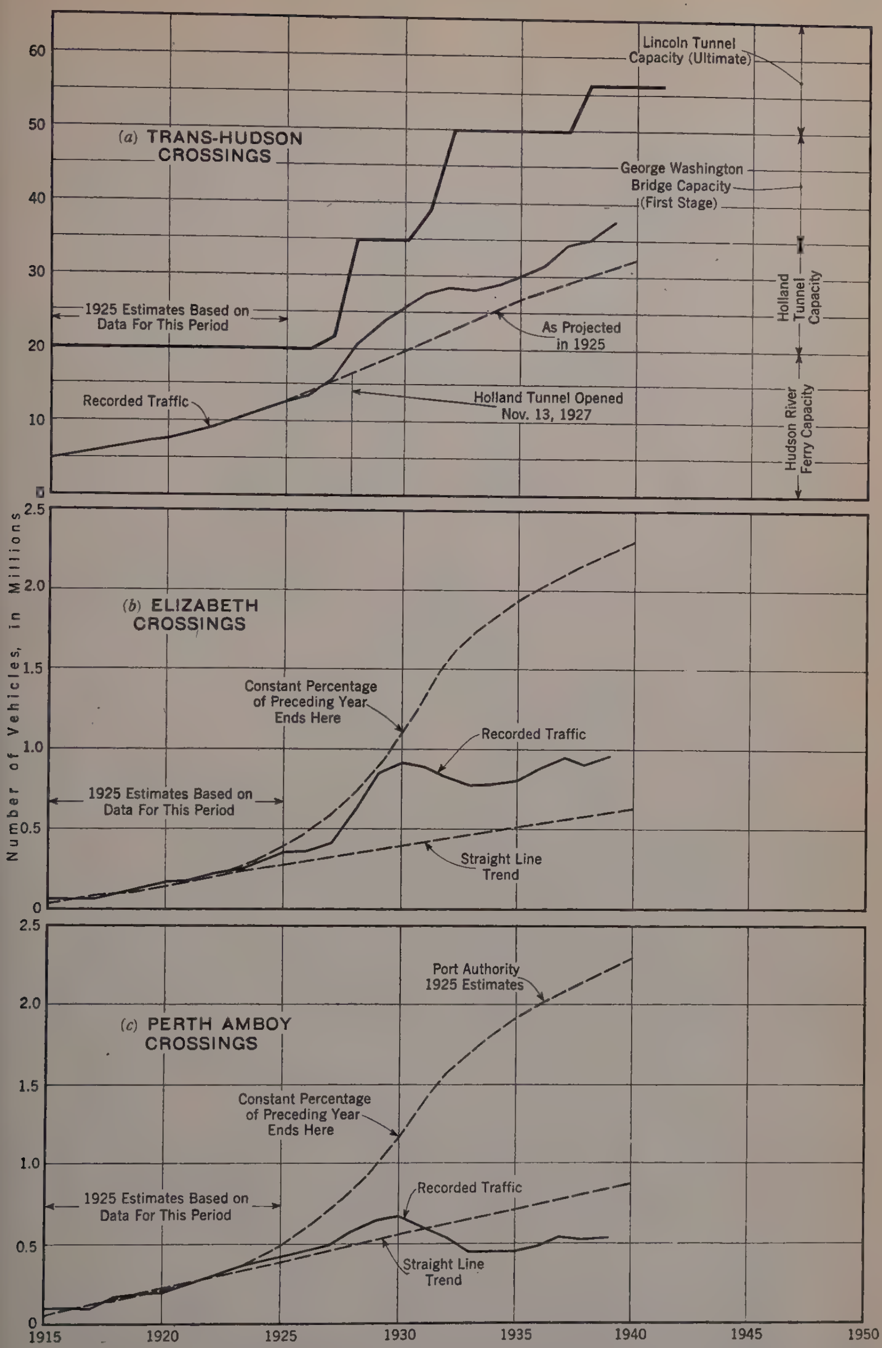

FIG. 6.—PROJECTIONS OF TRAFFIC FOR SELECTED CROSSINGS IN THE NEW YORK METROPOLITAN AREA

Motor Vehicle Ownership.—A much more intimate relation exists between growth of motor vehicle registrations in the areas tributary to the proposed crossing, and that of vehicular traffic changes. After all, a new car owner does mean a new motorist on some road or crossing, at some time. Besides, growth of motor vehicle registration also reflects growth of population completely. It must be emphasized, however, that before motor vehicle registrations may be used to forecast vehicular travel, future registrations themselves must be forecasted. Sometimes it is less difficult to forecast traffic itself than its determinant—vehicle registrations.

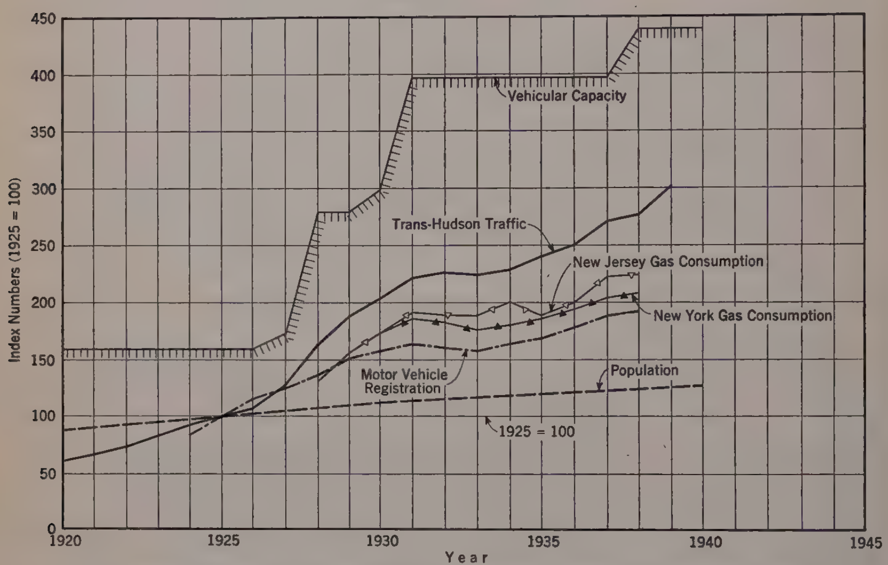

Fig. 7.—Annual Trends of Trans-Hudson Vehicular Traffic and Related Factors

Analyses of past trends in traffic and registrations indicated that traffic had expanded approximately in proportion to motor vehicle ownership. A comparison of trends of traffic and gasoline consumption, however, indicated that traffic expansion followed even more closely the expansion in gasoline consumption, because it reflected not only car ownership, but car usage as well (see Fig. 7).

Furthermore, in periods following the opening of new crossings, there occurred noticeable sudden "step-ups" in traffic volumes greater than those warranted by either the expansion in motor vehicle ownership or in gasoline consumption. These extraordinary volume jumps, largely attributable to increased vehicle usage or trip frequencies, were analyzed to determine their basic causes.

Economic Conditions.—Prior to 1930, the substantial upward trends prevailing in the two preceding decades largely offset any effect that economic conditions might have had on vehicular traffic expansion. Since 1930, rates of expansion have been stabilized and thus changes in traffic have reflected,

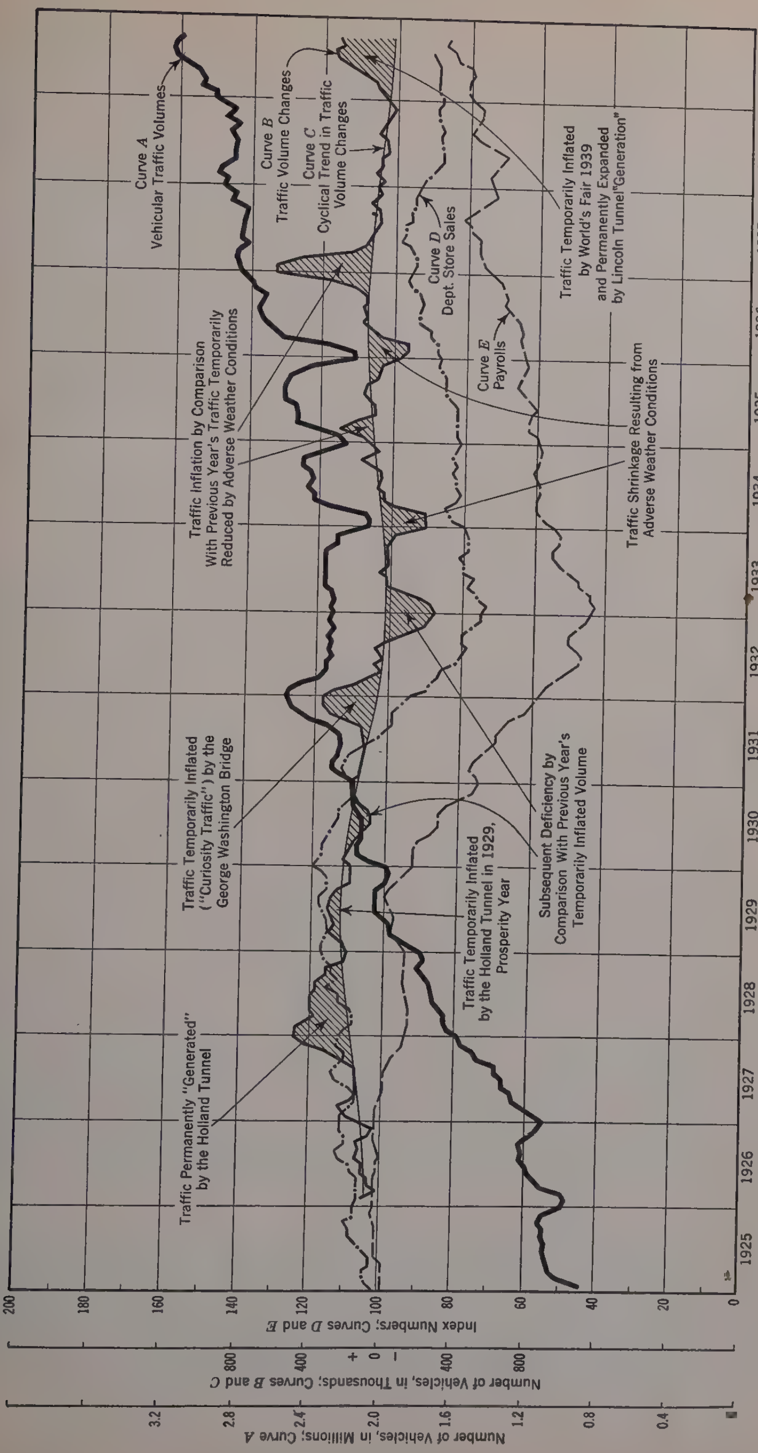

FIG. 8.—TRAFFIC OVER HUDSON RIVER CROSSINGS COMPARED WITH FACTORY PAYROLLS AND DEPARTMENT STORE SALES IN THE NEW YORK METROPOLITAN DISTRICT

more and more, the pronounced effects of economic conditions. Studies have shown that the rate of expansion of trans-Hudson vehicular traffic volumes, for example, was related to contemporary economic conditions, as reflected by factory payrolls and department store sales of the New York metropolitan district as will be seen from the curves in Fig. 8 which are defined as follows:

Curve (*A*)—vehicular traffic volumes—three month moving averages of seasonally adjusted monthly trans-Hudson vehicular traffic volumes.
Curve (*B*)—traffic volume changes—numerical changes in vehicular traffic volumes shown in Curve (*A*), from the same month in the previous year.
Curve (*C*)—trend in traffic volume changes—estimated cyclical trend obtained by spotting the crests and troughs in Curve (*B*) and fitting arcs of sine curves between them.
Curve (*D*)—department stores sales—three month moving averages of seasonally adjusted indexes (1923–1925 = 100) of department store sales in the New York Metropolitan District.[2]
Curve (*E*)—payrolls—three month moving averages of seasonally adjusted indexes (1925–1927 = 100) of New York City factory payrolls.[3]

At the time vehicular traffic volumes are projected into the future, it is nearly impossible to foresee the probable major changes in local economic conditions. These relationships, however, have served to explain the rate of traffic expansion in the past, and suggest that greater allowances in traffic trends will have to be made in the future for the effects of business conditions.

Extraordinary Traffic Expansion from Increased Vehicle Usage.—In the past, traffic has expanded approximately in proportion to motor vehicle ownership, except in periods following the opening of new crossings, when there were decided "step-ups" in traffic greater than warranted by the expansion in motor vehicle ownership.

The "step-ups" in travel attributable to increased vehicle usage (because of the added convenience of travel which followed the opening of the new crossing) could be measured by allowing for the expansion in motor vehicle ownership. In the case of traffic on the highways, vehicle usage could be measured by the average annual gasoline consumption per registered car in the entire state. In the case of vehicular crossings, a much more specific measure of usage was obtained in terms of travel frequencies, expressed as the annual number of cross-river trips per registered car, computed by dividing the annual traffic of groups of crossings serving tributary areas by the annual registrations in such tributary areas (that is, areas that furnished about 85% of the crossing's traffic). In both cases, allowance was thus made for changes in motor vehicle registrations, resulting in a measure of the effects of other factors on vehicle usage.

In the determination of the increased usage of a local facility, gasoline consumption data have not proved to be adequate for two reasons: The figures usually cover an entire state and, consequently, do not reflect local conditions; and these data have become available only in recent years.

[2] *Monthly Review of Credit and Business Conditions*—Federal Reserve Bank, New York City.
[3] *Monthly Survey of Current Business*—U. S. Dept. of Commerce.

Factors Affecting Increased Travel Frequencies.—Studies revealed that several factors were responsible for the "step-ups" that followed the opening of new crossings: (*a*) Removal of capacity limitations; (*b*) conversion of pedestrian, railroad, and trolley passengers to bus traffic; and (*c*) increased convenience of travel.

Removal of Capacity Limitations.—In the first place, where capacities of highway facilities were largely absorbed, such limitations were retarding normal expansion. Consequently, when these limitations were removed by the opening of new facilities, traffic volumes suddenly expanded to levels which they probably would have reached had there been no such limitations to retard "normal" expansion.

Increased travel frequencies, caused by the removal of capacity limitations of existing vehicular facilities, could be estimated from general analyses of past trends in travel frequencies under varying vehicular capacities, after measures of the annual working capacities of crossings had been established carefully (see Table 7).

TABLE 7.—MARGINS OF CAPACITY VERSUS TRAVEL FREQUENCIES VIA HUDSON RIVER CROSSINGS

Year (1)	Margin of capacity (percentages) (2)	Trips per vehicle registered[a] (3)	Year (1)	Margin of capacity (percentages) (2)	Trips per vehicle registered[a] (3)	Year (1)	Margin of capacity (percentages) (2)	Trips per vehicle registered[a] (3)	Year (1)	Margin of capacity (percentages) (2)	Trips per vehicle registered[a] (3)
1915	75.2	....	1921	58.4	20.2	1927[b]	25.2	14.9	1933	43.5	20.8
1916	72.4	....	1922	54.1	17.9	1928	40.8	17.5	1934	42.3	20.7
1917	69.3	30.2	1923	47.6	16.8	1929	31.8	18.4	1935	39.4	21.1
1918	67.0	28.3	1924	41.3	15.9	1930	26.4	19.3	1936	36.9	20.8
1919	64.1	24.9	1925	37.3	14.7	1931[c]	25.5	20.1	1937[d]	31.5	21.4
1920	61.9	22.0	1926	31.6	14.0	1932	42.9	20.7	1938	38.1	21.3

[a] Annual number of one-way trans-Hudson trips per vehicle registered in tributary counties east and west of the Hudson River. [b] Holland Tunnel opened November 13, 1927. [c] George Washington Bridge opened October 25, 1931. [d] Lincoln Tunnel opened December 22, 1937.

Bus Passengers.—In the second place, new bus lines were established, routed via the newer, faster crossings, and transporting former railroad, trolley, and ferry passengers (see Table 8). Increased travel frequencies, created by increased bus travel resulting from the shifts of railroad, trolley, and ferry passengers to interstate buses, could be determined by comparing rail and rapid transit fares with bus fares and travel times, allowing, in addition, for the extra convenience of door-to-door bus service as against station-to-station rail service.

Increased Convenience of Travel.—Simultaneously, additional "step-ups" in passenger car and truck traffic occurred, created by the extra travel conveniences afforded by these modern facilities. New feeder routes and faster crossings reduced travel time by from 15 min to more than an hour. These reductions in travel time, in effect, brought residential areas on one side of the river much closer to the working, shopping, and theatrical districts on the other

TABLE 8.—GROWTH OF TRANS-HUDSON BUS TRAFFIC, IN THOUSANDS

Year	Trans-Hudson ferries	Holland Tunnel[a]	George Washington Bridge[b]	Lincoln Tunnel[c]	All Hudson River crossings
1925	71.1	....	....	....	71.1
1926	154.7	....	....	....	154.7
1927	270.6	6.5	....	....	277.1
1928	238.1	232.1	....	....	470.2
1929	319.4	356.9	....	....	676.3
1930	314.7	462.4	....	....	777.1
1931	303.3	465.0	27.7	....	796.0
1932	163.6	390.0	238.4	....	792.0
1933	136.1	379.9	368.1	....	884.1
1934	145.0	344.1	439.3	....	928.4
1935	143.1	367.9	507.1	....	1,018.1
1936	129.8	378.3	592.7	....	1,100.8
1937	150.9	331.6	643.3	3.7	1,129.4
1938	61.4	307.3	633.5	186.1	1,188.3
1939	....	323.4	661.4	473.7	....

[a] Holland Tunnel opened November 13, 1927.
[b] George Washington Bridge opened October 25, 1931.
[c] Lincoln Tunnel opened December 22, 1937.

TABLE 9.—VEHICULAR TRAFFIC "GENERATION" EXPERIENCE FACTORS FOR SELECTED TOLL CROSSINGS

New crossings	Opening date	Competing crossings traffic[b]	"Generated" Traffic Expressed in Terms of:		Percentages of:			Average toll on new crossing, in cents
			Estimated annual volume of generated traffic	Annual toll cost, in thousands of dollars	Traffic diversion[b]	Competitive crossing traffic[f]	Crossing's traffic in first year[g]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(a) Crossings in the Port of New York								
Holland Tunnel	November 13, 1927	11,686,300	4,250,000	2,189	125	39.3	55.5	51.5
Goethals Bridge	June 29, 1928	409,973	365,422	190	187	89.1	64.5	52.0
Outerbridge Crossing	June 29, 1928	480,751	184,632	96	58	38.4	36.5	52.0
George Washington Bridge	October 25, 1931	4,987,000	2,225,000	1,168	65	45.0	39.5	52.5
Bayonne Bridge	November 15, 1931	583,160	336,420	173	244	57.7	70.9	51.5
Triborough Bridge	July 11, 1936	12,214,000	5,115,000	1,329	105	42.0	51.0	26.0
Lincoln Tunnel	December 22, 1937	5,245,000[c]	1,190,000	655	195	22.7	66.1	55.0
(b) Crossings Elsewhere in the United States								
Philadelphia-Camden Bridge	June 30, 1926	5,600,000	4,446,000	1,300	131	80.0	56.7	28.1
Carquinez Bridge	May 21, 1927	625,000	389,000	350	53	62.0	34.9	90.0
Tacony-Palmyra Bridge	August 15, 1929	376,228	875,900[d]	315	234	232.8[d]	70.0	36.0
Oakland Bridge } Golden Gate Bridge }	November 12, 1936 May 27, 1937	6,087,000	8,798,000	4,399	239	144.5	70.5	50.0
(c) One Crossing in Europe								
Queensway Vehicular Tunnel[a]	July, 1934	1,150,334	1,971,763	647[e]	145	171.4	59.1	55.5

[a] Between Liverpool and Birkenhead, in England. [b] Competing crossings traffic in the year preceding the opening of the new crossing. [c] Three nearby ferries only. [d] Includes an amount diverted from the lower Delaware River ferries and Camden Bridge that was difficult to determine. [e] This is the equivalent of 129,445£ sterling, at the exchange rate of about $5.00 per £. [f] Percentage of traffic of competitive crossings for the year preceding the opening of the new crossing. [g] Percentage of the new crossing's traffic in the first year of operation.

side. This increased speed, convenience, and safety of the new routes stimulated passenger car travel. Increased truck movements followed in the wake of the greatly increased passenger car travel in connection with such consumer industries as bakeries, laundries, etc.

Traffic "Generation" Experiences of Toll Crossings.—To determine the factors affecting increased passenger car and truck travel frequencies, several approaches were used. The first approach was to bring together a summary of "generation" experiences for a number of toll crossings in the United States and one in Europe (see Table 9).

The traffic "generation" experiences of these crossings, in the first year of their operations, may be summarized as follows:

(*a*) The volume of traffic "generated" varied from 185,000 to 8,800,000 vehicles;

(*b*) Additional transportation cost, paid for in tolls for this extra travel, varied from about $96,000 to almost $4,400,000;

(*c*) For every one hundred vehicles diverted from existing facilities, from 53 to 244 additional vehicles were "generated";

(*d*) For every one hundred vehicles handled by competing crossings in the year preceding the opening of the new one, the new crossing "generated" from 38 to 233 additional vehicles; and

(*e*) For every one hundred vehicles handled by the new crossing in the first year of its operation, from 35 to 71 vehicles constituted traffic "generated" by the crossing.

In all cases, except one, the new crossing was a toll bridge or tunnel, and the old competitive crossings were invariably toll ferries. In one case (the experience of the Triborough Bridge) the crossing developed its "generated" traffic in competition with free bridges. It is of especial interest to note that this experience (that of a toll bridge in competition with free bridges) is strikingly similar to the experiences of the other toll crossings in competition with toll ferries.

As in the case of any other empirical experience factors, however, the query in connection with these cases naturally arises as to what extent they would be applicable to future crossings elsewhere. Consequently, further analyses were made of some of the causative factors responsible for "generation" of traffic by new crossings to allow for specific factors that might be peculiar to a future crossing.

Quantitative Measures of Causative Factors of Expansion in Travel Frequencies.—An examination of travel frequencies between nearby and distant areas revealed the fact that the closer the areas, the greater the intensity of travel. New crossings, with their new approach highways, in effect, brought traffic centers closer together, in point of time, and thereby increased travel frequencies.

A further comparison of travel frequencies via toll-versus-free crossings (after allowing for distance and time) revealed the fact that travel frequencies via toll crossings were lower than those via free crossings. Consequently, it became necessary to determine a single measure incorporating trip time,

mileage, and other route characteristics, as well as tolls. This was accomplished by expressing, in dollars and cents, not only tolls, but also trip time, mileage, and other route characteristics (the last, although difficult to evaluate, must be allowed for), and thus arriving at a single trip-cost factor. Trip costs thus determined accounted for variations in travel frequencies and yielded this general rule: The lower the total trip cost, the greater the travel frequency (see Fig. 9).

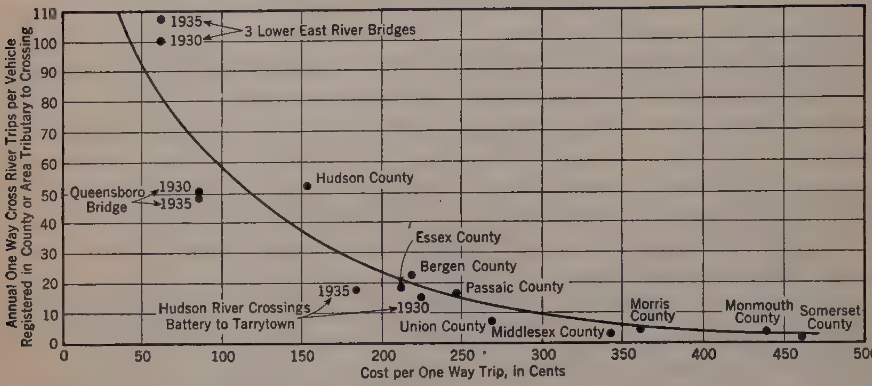

FIG. 9.—RELATION BETWEEN ANNUAL TRIP FREQUENCY AND THE COST OF A ONE-WAY CROSS-RIVER TRIP

It thus became evident that—(*a*) by reducing distance, running time, waiting time, or inordinate delays due to congestion, (*b*) by increasing the convenience of travel through the construction of additional crossings or new highways, or (*c*) by improving and integrating existing routes so as to eliminate confusing turns and "zigzags," thus making them more direct—travel frequencies could be expanded.

It should be emphasized, however, that the first bridge or tunnel which supplements a ferry service usually stimulates travel frequency substantially because, by eliminating the usual waiting time between ferry sailings, the bridge or tunnel thereby reduces travel time considerably. Subsequent competitive bridges or tunnels, on the other hand, usually will not stimulate travel frequency to the same extent as the previous crossings, even if increasing delays begin to be encountered at older fixed crossings, because these delays are usually not as great as those at intermittent crossings.

Conversely, by saddling traffic with additional costs—not only out-of-pocket costs, such as parking fees, mileage taxes, etc., but also subjective costs like economic losses imposed by customs barriers or other stringent governmental regulations, as well as undue delays on streets and highways, or, in general, increased difficulties and inconveniences of travel—travel frequencies can be expected to shrink.

The method of estimating changes in travel frequencies, therefore, resolves itself into determining trip costs properly before and after the opening of the new crossing, and consists of: (*a*) The inclusion of all the pertinent route characteristics; and (*b*) a proper evaluation of these characteristics on a dollar-and-cents basis (see Eq. 11 in the Appendix).

The present state of research indicates that a net change of one cent in trip cost (considering costs of all route characteristics) would produce a 1% change in travel frequency in the opposite direction. It is obvious that to sense the variations in the subjective values that motorists place on route characteristics at different times is an exceedingly difficult task, requires much constant research, and at present is at best an approximation.

Toll Reductions.—From the foregoing, it may be suggested that toll reductions would increase travel frequency and stimulate travel. Toll reductions probably do stimulate travel, but in many cases the stimulation, even together with diversions from competing crossings, is insufficient to prevent a decline in revenues. Consequently, toll reductions may seriously impair the financial stability of toll structures. On the other hand, it is of interest to note that, where bridges and tunnels have been opened, tolls on competing ferries have been reduced.

Trends in Travel Frequencies.—In addition to "step-ups" in travel frequencies, examinations of their trends for crossings in the New York district indicated that, in most instances, they have been increasing steadily and gradually over a period of years. This has been caused by the continuous program of highway expansion. Estimates of future trends in travel frequencies may be treated similarly to estimates of traffic trends (that is, either by projecting them on empirical or causative factor bases).

Conditions for Realizing Potential Traffic

Highways Must Support Crossing.—In weighing the relative merits of several alternate routes, motorists invariably consider them in their entirety from origin to destination. Therefore, the advantages of routes consist of two parts: (*a*) Those inherent in the crossing itself; and (*b*) those which are a part of the streets and highways along the routes between origin and destination, as well as the approaches and immediate approach connections to the proposed crossing. Consequently, as far as the motorist is concerned, a river crossing is only part of his complete route.

If the proposed crossing is to realize the potential traffic indicated by the technical methods described, therefore, the following conditions must be in existence when the crossing is opened to traffic:

(*a*) An adequate system of streets and arterial highways tapping its tributary areas and leading the traffic conveniently and directly to the crossing;

(*b*) Not only an adequate system of street and highway approaches in the vicinity of its plazas to handle its peak traffic, but one that is superior to those in the vicinity of the terminals of its competitor crossings; and

(*c*) Properly designed plazas on both sides of the river to keep traffic moving continuously via the crossing in peak periods.

Moreover, if the proposed crossing is to attract traffic from such of its tributary areas in which its own inherent advantages are nullified by street and highway conditions, it is obvious that such disadvantages must be removed by street and highway improvements. In addition, however, improvements must be so designed that they will not improve all other alternative routes

equally, but will give the routes via the proposed toll crossing decided time, distance, and convenience advantages over the then existing alternate routes and competitive crossings.

Plaza Designs.—In considering designs of plazas and approach connections, the following are a few of the conditions that may be encountered and which are of sufficient importance to require careful consideration.

Lane capacities of "signal-controlled" streets are reduced more or less drastically from what they would otherwise be under continuously moving traffic, whereas the crossing is designed for continuous traffic flow. The plazas serve as temporary storage areas, and convert intermittent traffic flow on to the plaza into continuous feeding of the crossing. In addition, the street lanes must also handle local traffic as well as crossing traffic. Each crossing lane, therefore, will usually require several street lanes, controlled by traffic lights, to which to deliver, or from which to be fed by, its traffic. Consequently, a careful examination must be made of the existing street layouts and of their effective capacities to handle their share of local traffic and at the same time, adequately, to diffuse and feed the traffic of the proposed crossing without limiting its capacity.

Thus, to provide sufficient capacity to feed the Lincoln Tunnel its traffic, and then diffuse it, it was necessary to build two new streets on Manhattan Island, one running north and south between Ninth and Tenth avenues, the other parallel between Tenth and Eleventh avenues, and between 34th and 42d streets. A similar plan is to be followed on the east side of Manhattan in connection with the plaza of the Queens Midtown Tunnel (see Fig. 10).

Another pertinent fact to consider is that traffic bound for a crossing may be fed into its approach streets via the immediately nearest intersection, controlled by a traffic light, from both intersecting streets, thus obtaining the benefit of practically the entire traffic-light cycle. On the other hand, traffic flowing away from the crossing can clear the next immediate light-controlled intersection only out of one of the two intersecting streets, and can thus utilize only part of the traffic-light cycle (assuming that right-hand turns are not permitted on the red light). Therefore, especially where only the smaller portion of the light cycle is available to diffuse the crossing's traffic, a special examination must be made of those streets to make certain that, in the aggregate, as well as in each of the four directions from the plaza, they have sufficient margins of capacity, above local traffic requirements, to carry peak crossing traffic without limiting the capacity of the exit lanes.

In addition to local traffic, parts of certain streets may be called upon to handle both entrance and exit traffic of the proposed crossing. It is essential to examine the probable peak traffic which they may be called upon to handle, and their adequacy to do so.

In congested business areas, furthermore, part of the crossing's traffic may approach the plaza from directions opposite to those of its points of origin. Thus, part of the passenger-car traffic may use an express shore drive, even at the expense of a "back-haul," to avoid the center-of-town congestion, and approach the crossing "from behind" its plazas. Truck traffic from waterfront terminal facilities may also approach the crossing in this way. Where such

conditions are encountered, adequate convenient connections to the waterfront areas and highways must be provided.

These and similar considerations usually require that standard estimates be prepared of: (*a*) Peak-hourly traffic on approach streets in every direction to and from the proposed crossing plazas; (*b*) gross-hourly capacities of each of the approach streets; (*c*) their present and future peak-hour local traffic; (*d*) their margins and capacity available for the traffic to and from the proposed crossing; and (*e*) the probable peak-hour surplus margins (or deficits) over the traffic requirements of the proposed crossing (see Fig. 10).

Furthermore, for the purpose of appraising the relative merits, from a traffic-capacity standpoint, of the many different designs of plaza and approach connections which are usually prepared and studied before the final design is adopted, a series of traffic flows corresponding to each design must also be prepared. With the aid of these traffic flows, it is possible to weigh alternative designs on a quantitative basis, and eventually to select a plaza design and an adequate system of street approaches, acceptable not only from a traffic but also from other equally essential points of view.

Necessity for and Difficulties Encountered in Checking Traffic Estimates

Realized Versus Anticipated Traffic.—In connection with the planning of free highway facilities, traffic estimates, if prepared at all, are seldom checked against traffic realized, once the facility is open. It is more than likely that such estimates would usually be exceeded, since the new highway would be so far superior to those existing and, costing nothing to use, would attract and create large volumes of traffic. In the case of vehicular toll facilities, however, traffic may be larger or smaller than estimated. In either event, a check-up of estimated versus realized traffic would be necessary.

If traffic volumes are greater than anticipated, they are often so much larger that a very substantial percentage of the capacity of the new facility is absorbed at once. If traffic volumes remain at high levels, plans for the expansion of the new facilities or the construction of new facilities are soon begun. If the traffic is smaller than anticipated, check-ups are necessary to determine the causes for the failure to attract the traffic forecasted, and to stimulate patronage of the new facility through advertising programs and proper "signing" of the highway routes leading to and from the new facility.

Total Traffic "Reservoir" Versus Divertible Shares.—In checking estimates versus realized traffic, it is generally found that there is least divergence in the data for the total traffic in the "reservoir" (see Fig. 6); but that fact is only of academic interest. On the other hand, the real differences occur in the estimated distribution of traffic among the competitive facilities; and that is a practical matter. It is not enough that the total traffic estimated in the "reservoir" be there—it is also essential that it be on the facility that was financed and constructed to attract the volume estimated. It is necessary, therefore, to explain differences in the distribution of traffic among competitive facilities and suggest means for attaining the distribution estimated.

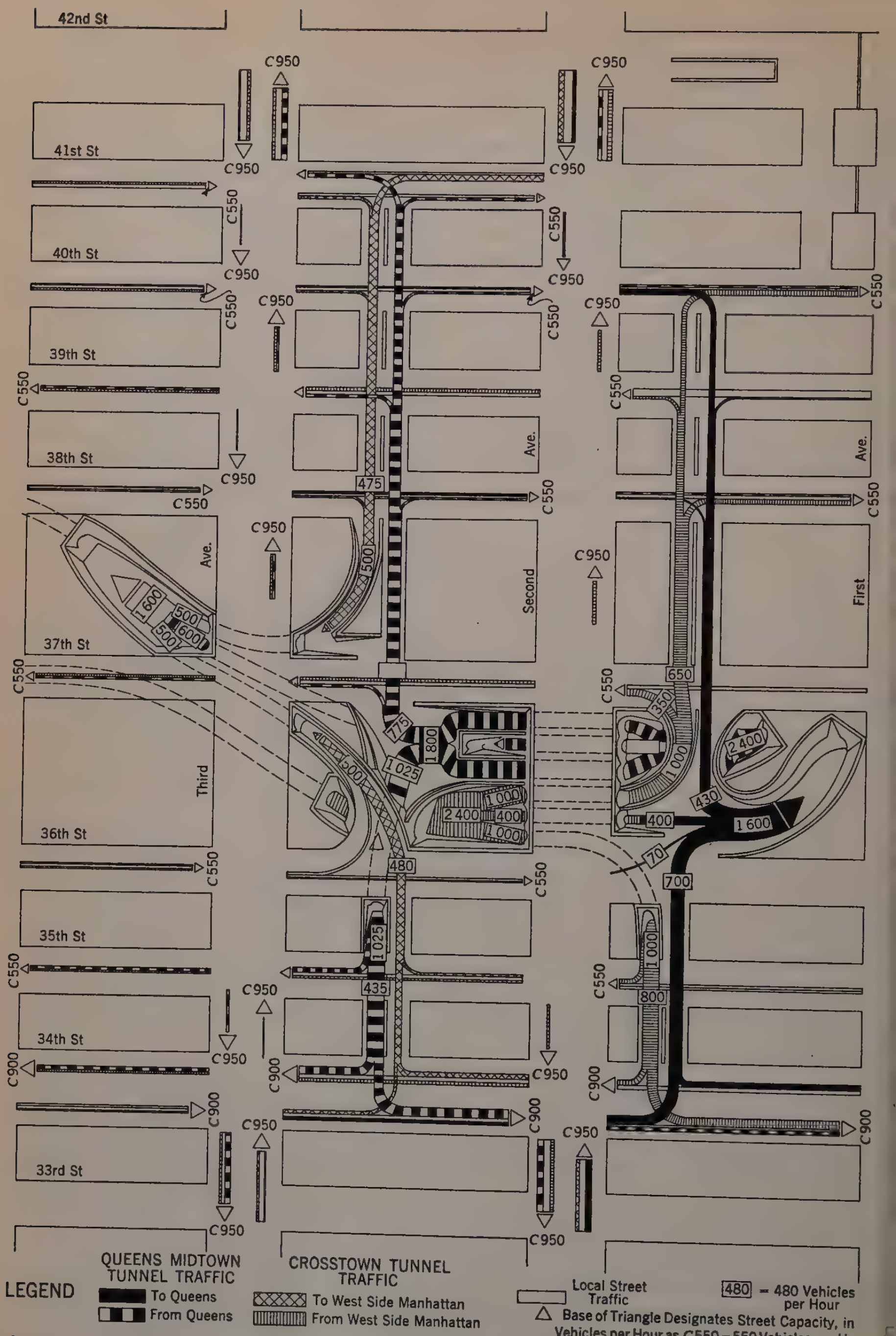

FIG. 10.—QUEENS MIDTOWN (N. Y.) AND MANHATTAN CROSSTOWN TUNNELS AND LOCAL STREET TRAFFIC FLOWS IN THE VICINITY OF THE JOINT MANHATTAN PLAZAS DURING THE MORNING PEAK HOUR, 8:00 A.M. TO 9:00 A.M.

Subsequent Changes Affecting Estimates.—In the period between the preparation of traffic estimates and the opening of the new facility, radical changes occur in the trends of the several elements of potential traffic, and radical traffic shifts occur among the vehicular facilities from which traffic was to have been diverted to the new facility. Thus, migrations of population between areas tributary and non-tributary to the proposed crossing, important changes in general economic conditions, and changes in the tolls on existing competitive crossings (usually reductions), as well as the construction of new and superior express highway routes, may all affect its potential traffic greatly—either favorably or adversely.

Consider, for example, the Henry Hudson Bridge in New York City: It was evident at the end of its first year of operation (December 31, 1937) that the superior express parkway that feeds it from the north and south would actually supply far more than its estimated traffic, that its capacity would be absorbed, and that a new additional level would be needed soon. This level was financed immediately and construction begun, and it was opened in May, 1938, or 10 months from the time of financing.

On the other hand, consider the Lincoln Tunnel in New York City. During the construction of the tunnel, the greater part of the anticipated expansion of midtown ferry traffic, which forms its divertible traffic "reservoir," was absorbed by the Holland Tunnel and George Washington Bridge because of the modern express highway approaches serving them (the Pulaski Skyway (New Jersey), leading to the Holland Tunnel; Routes 4 and 6 (New Jersey), leading to the George Washington Bridge; and the West Side Highway and Henry Hudson Parkway, on the New York side). These highways brought these two facilities closer, in time, to midtown Manhattan and Long Island (that is, the highways increased their "ratings" along these "lines of travel" over their original "ratings," at the time of preparation of estimates), although longer in distance than the more direct routes via the midtown ferries. It is thus more difficult for the Lincoln Tunnel to divert traffic from the Holland Tunnel and George Washington Bridge than if that same traffic had remained on the midtown ferries.

In the second place, the nearby competitive ferries, having written off the greater portion of their investments in facilities and equipment, have taken advantage of this financial condition to reduce their tolls (that is, competitive ferries have increased their ratings though reduced tolls) and thus made it more difficult for the tunnel to divert their present traffic.

In the third place, the tunnel was opened in a period of economic recession, which condition contributed further to the difficulty of the tunnel's diverting traffic from the cheaper, although slower, competitive facilities (time is worth less in a depression period).

In the fourth place, as has been mentioned previously in this paper, from the standpoint of the motorist who is to patronize the new facility, it is the advantages of the entire route from origin to destination which determine the choice of route and not merely the advantages and novelty of a new crossing. Two important feeder highways in its own tributary area, and approach con-

nections thereto, were incomplete when the tunnel was opened, whereas traffic estimates had been predicated on their completion. Consequently, its realized traffic naturally fell short of estimates. This lack of approach connections was remedied partly by the opening of the express approach connections through Union City, N. J., on June 30, 1939. As motorists become aware of these connections, there will be a shift from alternate routes to the tunnel. That such shifts are now in progress is strikingly illustrated in Table 10, which indicates that, as the tunnel's supporting highways come up to those contemplated in original estimates, its realized traffic gradually approaches original estimates.

TABLE 10.—PROGRESS OF REALIZED LINCOLN TUNNEL[a] VEHICULAR TRAFFIC TOWARD TRAFFIC ESTIMATES

Date	Description	Monthly traffic
May, 1938	Spring month close to average month of year; approach connections still under construction	151,653
October, 1938	Fall month close to average month of year; "loop approach" in Weehawken, N. J., opened to traffic October 15, 1938	172,154
May, 1939	Service streets paralleling express highway approach under construction opened in stages	203,046
October, 1939	Express highway through Union City, N. J., opened June 30, 1939	311,648
December, 1939		302,260 [b]
Average month, 1939	As originally estimated by methods described in this paper	437,500

[a] Opened to traffic December 22, 1937. [b] Seasonal drop.

Thus, it will be seen that, in connection with the operation of new toll crossings, it is necessary to analyze, currently, traffic volumes of the entire "reservoir," segregated as to the sources of potential traffic of the new facility on a number of different bases, such as: By competing crossings, by days of the week, by types of vehicles, and by areas of origin or destination. It is also necessary to follow such changes as reductions in tolls of competing crossings; construction of new highways, particularly in areas immediately tributary to the crossing; changes in the economic situation and in parking regulations; and a host of other new, spasmodic, but related factors that affect the current volumes of the new crossing. All of these data are necessary in order to explain and to suggest means of meeting the changes which had taken place subsequent to the preparation of the original estimates.

Measuring Changes in Geographical Distribution.—Where current annual traffic data are compiled for all alternative routes, it is possible to check, at once, the changes in the total annual volumes that have occurred on competing alternate routes between the time of preparation of the estimates and the opening of the facility. It is more difficult, however, to determine the differences in the trends of traffic volumes in areas tributary, as compared with those not tributary, to the proposed crossing—especially since most crossings serve both types of areas. Traffic volumes over a period of years for individual areas practically do not exist. Fortunately, however, in connection with studies of the George Washington Bridge and the Lincoln Tunnel, and with the aid of

WPA personnel in recent years, origin and destination data were collected and compiled in 1925, 1930, and 1935. These data permit one to obtain a glimpse of the radically different rates of expansion that occur in traffic volumes originating in different counties, some tributary, and some not tributary, to the new facility (see Fig. 5).

Measuring Actual Diversions.—Moreover, traffic volume data segregated as between passenger cars, trucks, and buses are rare, and only in recent years have sample traffic counts permitted such figures to be compiled by The Port of New York Authority. Where current traffic volume data for crossings competing with the new one are available, it is possible, by a simple comparison of the volumes before and after the opening, to determined the traffic decline on these crossings. However, such traffic declines may be the result not only of (*a*) diversions to the new crossing, but of (*b*) shrinkages due to an economic depression or recession, or (*c*) shifts to facilities other than the new one, which have put toll reductions into effect, and (*d*) gains offsetting diversions stimulated by toll reductions and other traffic stimulating factors. It thus becomes practically impossible, at times, to segregate diversion losses from other concurrent losses, especially where successive changes in the policies of the operators of competing crossings make for radical shifts back and forth among competitive routes.

Distinguishing Current Fluctuations from Permanent Trends.—Thus, the conditions that arise upon the opening of the new facility are not only radically different from those upon which diversion estimates were predicated, but the current changes themselves make it exceedingly difficult, at times, to put the estimating methods to a practical test until a year or two have elapsed since the opening of the facility. At this time, other fundamental changes have taken place which must be considered. Looking back, these shifts may be ascertained more accurately but, for the operators, "post mortems" are cold and academic.

"Generations" Versus Diversions.—It is also interesting to make this observation: Although the anticipated traffic volume to be "generated" is invariably a highly intangible item, and although it usually forms a substantial share of the total potential traffic of a proposed crossing, this element, nevertheless, has been realized to a larger extent than other elements of potential traffic, such as diversions from competitive routes, which appear to be more tangible, although apparently more elusive in their realization. It is fairly definite, however, that new facilities, which make a contribution to speed, convenience, and safety of travel, can usually count on a substantial volume of "invisible" traffic ready to avail itself of the advantages they offer.

Checking Estimated Directional Traffic Flows.—Estimates made of the directional distribution of the traffic of the new facility, on its plazas and along its approach connections to and from the four points of the compass, can usually be checked only in a very limited area immediately adjacent to the new crossings. Beyond those points, local traffic usually merges with that of the new facility, and it becomes exceedingly difficult to differentiate between crossing and local traffic. Consequently, if congestion should occur in the vicinity of the new crossing, it becomes difficult to determine to what extent it is caused by

the traffic of the new facility, and to what extent by the local traffic, which use the approach connections jointly.

These are only a few of the difficulties that one encounters in comparing traffic estimates with traffic realized in an effort to determine from past experiences the various and devious pitfalls into which the traffic estimator may fall, and to determine ways and means of realizing the traffic which the estimating methods discussed may indicate.

Conclusions

To those who have occasion to prepare traffic and revenue estimates of proposed self-liquidating vehicular toll facilities, the summary of the determining factors shown in Fig. 11, and the following directions, may be helpful.

Traffic Estimates a Balanced Appraisal of Economic Soundness.—The ultimate function of traffic and revenue estimates is to furnish the executive with a nicely balanced appraisal of the economic soundness of the proposed vehicular toll crossing. If the crossing is found to be economically practicable, the supporting estimates must be such as to inspire confidence that motorists will patronize the facility in sufficient volume to preclude financial embarrassment of the facility when it opens, and also, incidentally, prevent other communities from financing meritorious projects. If the proposed facility is proved economically unsound, the supporting estimates must be such as to assure the executive that the community is not being deprived of a facility it can well afford and would patronize, and that a loan is not being rejected on a project that may be a good financial risk. In short, estimates must be such as to aid the executive in deciding whether the motoring public shall enjoy a new traffic facility, or whether the investing public shall be saved the grief of an unsound investment.

It is obvious that traffic and revenue estimates that are too optimistic must be avoided. On the other hand, ultraconservative estimates are equally undesirable. Thus, if traffic and revenue estimates of a proposed facility are so conservative as to contemplate no expansion in the future, for example, they may be interpreted in two ways:

(1) The facility has reached its zenith and must inevitably enter a state of decline—in which case, why should the banker underwrite it? Or

(2) The task of peering into the future is too hazardous for the engineer—in which case the banker must set the price of the funds high enough to cover the heavy risk, thereby probably destroying its chance of being financed.

If engineers are to benefit from the construction of vehicular toll facilities that are sufficiently essential to the traveling public to be practicable economically, they must be willing to give serious thought to the economic as well as the engineering phases of vehicular facilities. Proof of the economic practicability of these facilities constitutes the inertia to their initiation. On the other hand, their financial success "generates" the demand for other similar

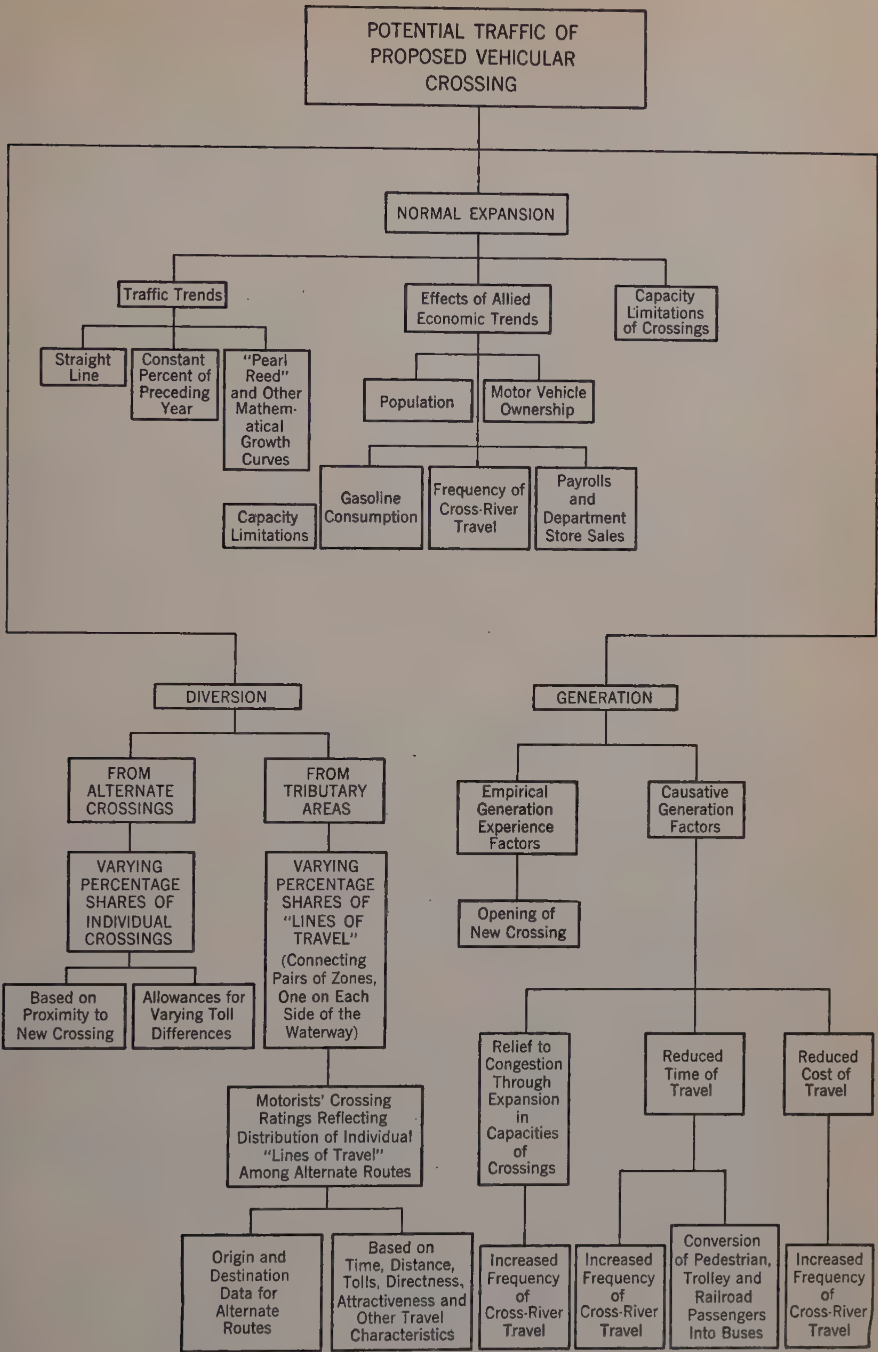

FIG. 11.—OUTLINE OF PROCEDURE FOR A TRAFFIC SURVEY

facilities. The engineer has made, and can continue to make, two effective contributions to promote the construction of vehicular toll crossings:

(1) He must design facilities which, not only from an engineering standpoint but especially from a traffic standpoint, will be so far superior to existing competitive highway facilities that they will attract to themselves the bulk of the motorist patronage traveling between their tributary areas, as well as create new patronage, thus assuring that such projects will be financial as well as engineering successes; and

(2) Although the bulk of the traffic of the proposed toll crossing may not be presently "visible," he must discover such sound methods of measuring its potential traffic as will inspire the bankers' confidence whenever his prognostications point to its economic soundness and financial success.

Acknowledgment

The writer is indebted to The Port of New York Authority for the privilege of using the wealth of data collected in connection with its studies of the economic practicability of its vehicular crossings; to the WPA (Project No. 665–97–3–65) for assistance in the collection, compilation, and analysis of supplementary data; and to S. Woolman, assistant engineer, WPA, for aid in analyzing the basic data.

APPENDIX

Mathematical Formulas for Developing Estimates of Diversions, "Generations," and Expansions in Traffic Volumes Handled by Vehicular Traffic Facilities

The following formulas describe, mathematically, the basic factors that affect, and the manner in which they bring about, changes in traffic volume. These formulas indicate the type of motorist mass reaction to changes in the underlying factors that affect traffic volumes. The values of the "constants" in these formulas must be determined, in every specific case, on the basis of data collected for the express purpose of determining these "constants" for the specific location and year.

The estimates resulting from the use of these formulas are "first approximations," rather than final estimates, and should be used only as aids to engineering judgment. Engineering judgment may point to the existence of "preference" or "prejudice" factors which are not measurable, but which are nevertheless real, and must be inserted in the formulas as additional factors likely to reduce or augment the volumes determined as "first approximations." The formulas are such as to allow for the insertion of judgment factors.

Notation

The letter symbols used in the formulas of this appendix are defined as follows:

a = a constant annual increment of traffic growth (Eq. 6);

C = cost; weighted average, one-way cost of a trip across the river (considering travel-time, distance, tolls and other route characteristics) along any given "line of travel," via all alternate crossings:

- C_0 = base cost (Eq. 11);
- C_1 = cost before the opening of a new crossing (Eq. 11);
- C_2 = cost after the opening of a new crossing (Eq. 11);
- ΔC = net travel cost difference between the proposed crossing and the "standard" crossing (Eq. 7);
- ΔC_d = monetary evaluation of the mileage difference, in cents per mile (Eq. 7);
- ΔC_r = monetary evaluation of the running-time difference, in cents per minute (Eq. 7);
- ΔC_w = monetary evaluation of the waiting-time difference, in cents per minute (Eq. 7);
- ΔC_p = "Prejudice" or "preference" cost difference of the convenience, attractiveness, directness, scenic values, and other directly immeasurable factors, in cents (Eq. 7).

ΔD = distance difference via crossing, compared to that via the "standard" crossing, in miles (Eq. 7);

d = discount factor by which unity (1) is reduced for every cent of excess travel cost over that via the "standard" crossing (Eq. 6);

F = frequency of travel, in one-way, cross-river trips per year:

- F_0 = frequency at a base cost of travel (Eq. 11);
- F_1 = frequency before the new crossing is opened (Eq. 11);
- F_2 = frequency after the new crossing is opened (Eq. 11);
- ΔF = percentage change in travel frequency, corresponding to a change in travel cost, following the opening of the new crossing (Eq. 11);

m = ratio of the margin of capacity to the annual volume in the base year (Eq. 10);

N = number of years between the base year and any given year (Eqs. 8, 9 and 10);

p = percentage of traffic along a "line of travel" handled by a given crossing (Eqs. 3 to 5);

q = premium factor; a percentage by which travel frequency is increased for every cent of reduction in the average trip cost (Eqs. 11);

R = annual rate of change in the ratio of margin of capacity, m (Eq. 10);

r = estimated "rating" of any proposed crossing (or any existing crossing whose factor ΔC has changed), based on its travel-cost differences from the standard competitive crossing; $r_1, r_2, \cdots r_n$, = the ratings of crossings $1, 2 \cdots n$; the rating of a "standard" crossing is unity (1);

ΔT = toll difference via any crossing compared to that of a "standard" crossing, in cents (Eq. 7);

t = time, in minutes:

Δt_r = running-time difference via any crossing compared to that via the "standard" crossing (Eq. 7);

Δt_w = waiting-time difference via any crossing compared to that via the "standard" crossing (Eq. 7);

V = annual volume of traffic:

V_0 = annual volume in the base year (Eqs. 8 and 9);

V_n = annual volume in any given year (Eqs. 8, 9, and 10);

V_m = maximum annual volume determined by the aggregate capacity of all facilities (Eq. 10);

ΔV = annual percentage traffic increase over preceding year;

v = the volume of traffic along a "line of travel" handled by all competing crossings (Eqs. 1 and 2):

v_n = volume handled by crossing (n) (Eqs. 1 to 5);

v_p = volume handled by the "standard" crossing (p);

$v_1, v_2, \cdots v_n$ = volume handled by crossings $1, 2, \cdots n$.

Percentage Shares and "Ratings" of Existing Crossings for Any Given "Line of Travel" Based on the Volumes Handled by Each Crossing

By definition (see "Notation"):

$$p_n = \frac{v_n}{v} \qquad (1)$$

but,

$$v = v_p + v_1 + v_2 + \cdots v_n \qquad (2)$$

Therefore,

$$p_n = \frac{v_n}{v_p + v_1 + v_2 + \cdots v_n} \qquad (3)$$

In Eq. (3) dividing numerator and denominator by v_p:

$$p_n = \frac{\frac{v_n}{v_p}}{\frac{v_n}{v_p} + \frac{v_1}{v_p} + \frac{v_2}{v_p} + \cdots \frac{v_n}{v_p}} \qquad (4)$$

By definition, $r_n = \frac{v_n}{v_p}$; $r_1 = \frac{v_1}{v_p}$; $r_2 = \frac{v_2}{v_p}$; $\cdots r_n = \frac{v_n}{v_p}$. Therefore,

$$p_n = \frac{r_n}{1 + r_1 + r_2 + \cdots r_n} \qquad (5)$$

"Rating" of a Proposed Crossing Based on Cost Differences from "Standard" Competitive Crossings

Referring to "Notation":

$$r = (1 - d)^{\Delta C} \qquad (6a)$$

or, approximately,

$$r = 1 - \Delta C\, d \qquad (6b)$$

Evaluation of Travel Characteristics of the Proposed Crossing

Referring to "Notation":

$$\Delta C = \Delta t_r\, \Delta C_r + \Delta t_w\, \Delta C_w + \Delta D\, \Delta C_d + \Delta T + \Delta C_p \qquad (7)$$

Mathematical Expressions of Traffic Growth

Growth consisting of constant annual numerical increments (straight line) may be determined by:

$$V_n = V_0 + a\, N \qquad (8)$$

Growth consisting of constant annual percentage increments (logarithmic straight line or compound interest curve) may be determined by:

$$V_n = V_0\, (1 + \Delta V)^N \qquad (9a)$$

or,

$$\log V = \log V_0 + N \log (1 + \Delta V) \qquad (9b)$$

Growth consisting of variable annual increments dependent on available margin of capacity ("Pearl Reed" curve) may be determined by:

$$V_n = \frac{V_m}{1 + m\, R^N} \qquad (10)$$

Expressions for Changes in Travel Frequency Following the Opening of a New Crossing or New Highway Facilities

Referring to "Notation":

$$F_1 = F_0\, (1 + q)^{C_0 - C_1} \qquad (11a)$$

$$F_2 = F_0\, (1 + q)^{C_0 - C_2} \qquad (11b)$$

and,

$$\frac{F_2}{F_1} = \Delta F = (1 + q)^{C_1 - C_2} \qquad (11c)$$

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

SEALING THE LAGOON LINING AT TREASURE ISLAND WITH SALT

By Charles H. Lee,[1] M. Am. Soc. C. E.

Synopsis

Fine-textured earth materials, such as clay or clayey sand and gravel, are potentially impervious to water. Sometimes, however, experience in the use of such material for watertight construction has been disappointing. In some cases, clay membranes placed with the greatest of care have been found to be semi-pervious and have failed to perform the function for which they were designed. This paper describes such an experience at Treasure Island, the site of the Golden Gate International Exposition in San Francisco Bay, California, and the simple method in which the defect was remedied by utilizing an electro-chemical phenomenon of colloidal clay.

The paper describes the novel method by which the 10-in. clay lining of the bottom of the seven-acre fresh-water Lagoon was sealed by a priming of salt water pumped in from the bay. The lining was compacted by use of a 14-ton flat roller, but with comparatively low average density and inclusion of considerable air. Initial seepage loss from fresh water in a test pool was 1.00 in. per day. This was reduced to 0.10 in. per day by the salt water treatment described herein.

Definitions

Adsorption.—When a gas or solution is brought into contact with a very finely divided or porous material, the pressure of the gas or the concentration of the solution decreases, the gas or solute becoming attached to the surface of the solid. This is the electro-chemical phenomenon known as "adsorption." The gas or solute can be removed from the surface of the solid by exhaustion, heating, or washing. The efficiency of an adsorbent depends largely upon its specific surface (area of surface per unit mass). The forces of adsorption correspond closely with chemical forces or affinity.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 15, 1940.

[1] Cons. Engr.; formerly, Chf. of Div., Water Supply and Sanitation, Dept. of Works, Golden Gate International Exposition, San Francisco, Calif.

Adsorption Compounds.—The union of atomic or ionic aggregates or mobs in indefinite proportions, under the influence of the forces of adsorption, produces "adsorption compounds." They result from the uniting of particles which have not attained proper subdivision and proximity through solution, fusion, ionization, or pressure to permit the combination of isolated atoms or ions in true chemical proportions. In soil they occur as colloidal soil particles acting as anions (negatively charged) and holding upon their surfaces, by adsorption, ions acting as cations (positively charged) which have been drawn from the surrounding soil solution.

Base Exchange.—A chemical reaction in which two bases or metals exchange places (one being ionized in solution and the other chemically combined as a molecular constituent of an insoluble solid) is known as "base exchange." The process is reversible, the direction depending upon the molecular concentration of the solution. The phenomenon is illustrated by the action of natural zeolites such as sodium silicate in removing calcium and magnesium from hard water which passes over them and giving up sodium to the solution. In soils containing adsorption compounds it may be illustrated by the replacement of calcium ions attached to colloidal soil particles, by sodium ions from the surrounding soil solution.

Calcium Clay and Sodium Clay.—In "calcium clay," calcium predominates, and in "sodium clay," sodium predominates, among the cations resulting from the dissociation of dissolved salts in the soil-water solution. The finer clay and colloidal particles in such a material, acting as negatively charged anions, hold upon their surface the calcium (or sodium) and other positively charged cations adsorbed from the soil solution.

A calcium clay has low sticky properties and the colloidal particles are well flocculated or grouped into aggregates as flocs or crumbs which act somewhat like grains of silt or fine sand and impart a semi-pervious character to the mass. In agricultural soils and fresh-water sediments, calcium is normally the predominant exchangeable ion associated with the clay.

A sodium clay, on the other hand, is very sticky, and with a soil solution of low concentration its colloidal particles are well deflocculated or dispersed into the ultimate minimum size, rendering it impervious to the transmission of water. It is the characteristic clay in salt and alkaline soil and in sediments formed under marine, estuarine, and deltaic conditions.

Methods of Rendering Clay Impervious

Impervious clay beds are of frequent occurrence in nature. The most common physical processes by which such condition is attained are: (*a*) By compaction to a high state of density under weight of superimposed material or long-continued pressure; and (*b*) by complete dispersion of clay and colloids through washing with sea water or impregnation with alkali salts.

Utilizing the first of these natural processes, man has rendered clay impervious, from time immemorial, by the process of puddling, which consists of working clay, or a graded mixture of clay, sand, and gravel, with water to render it compact and impervious. The material receives a preliminary grinding in a pug mill or otherwise the solid particles are thoroughly mixed

with water, followed by tamping, ramming, or kneading, as with the trampling of animals, so as to produce high density in the mass. Puddling brings the particles into closer contact, by expelling air and interlocking the granular particles, to the point where the forces of molecular attraction are brought into play. Colloidal materials, including the colloidal aggregates of calcium clays, are rearranged and compacted, thus filling the inter-granular voids and rendering the material permanently impervious to water. With modern construction methods the tamping and kneading action required in puddling is obtained mechanically by use of the sheepsfoot roller.

The second natural process by which clays are made impervious has been recognized only in recent years, and its practical application in construction is still in the experimental stage. The process consists of treating the clay with a sodium salt such as NaCl, or Na_2CO_3, to render it more cohesive and impervious to water. This is accomplished through the chemical process of base exchange by which the calcium ions attached to colloidal particles are replaced by sodium ions from the soil solution. After leaching with fresh water to remove excess sodium ions from the soil-water solution, the colloidal aggregates disperse, changing the physical properties of the soil and filling the voids with a sticky colloidal gel which is impervious to water.

This treatment is applicable, with most success, principally to soils and sediments containing calcium clays in which the "silica sesquioxide ratio" or ratio of silica to iron plus aluminum oxide is high (2 to 7). Such soils are predominant in cool, humid localities such as those in the northern and central regions of the United States and along the Pacific Coast. In the warm humid southeastern states and in the humid tropics, soils with low ratio (2 or less) are common, and sodium treatment has little effect upon physical properties. Residual soils derived from certain igneous and volcanic rocks also show little or no change with sodium treatment.

Sodium treatment is the reverse of the process commonly used in agriculture to improve the working qualities of a heavy sticky soil. For the latter purpose calcium is added to the soil in the form of lime ($CaCO_3$), or gypsum ($CaSO_4$). The purpose is to replace the sodium with calcium and thus soften the soil and render it more pervious by giving it a crumby structure. A familiar example of the application of this process on a large scale occurs in the land reclamation which has been in process for many centuries along the north coast of Holland. The silt deposited by the sea is allowed to accumulate until there is a sufficient area to justify the building of a dike to prevent further incursions of the sea. The finer-textured material which is deposited, having been in contact with sea water, is essentially a sodium clay, but in addition there is a large percentage of calcium carbonate particles. After a lapse of a year or two, surplus sea water is mostly removed in the drainage water derived from rainfall and river overflow. The calcium carbonate then reacts with the sodium clay, gradually converting it into a calcium clay which is capable of successful agricultural production.

Although calcium treatment is common in agricultural practice, sodium treatment, its reverse, has had only experimental application in engineering and construction practice. The first instance, which attained widespread notice, was

the use of soda ash in rendering watertight the cutoff wall in the Alexander Dam on the Island of Kauai, Hawaii.[2] This application was entirely successful.[3] More recently salt has been used in road construction as an admixture to stabilize and render impervious earth road surfaces.[4] This process is still in the experimental stage but results appear favorable. The use of sea-water to seal the clay lining of the Lagoon at Treasure Island was probably the first successful use of salt in hydraulic construction. The work was done under the writer's direction, drawing upon a background of investigation and experimental work beginning in 1930.

Treasure Island Lagoon

An important feature of the landscaping at Golden Gate International Exposition on Treasure Island was the Lagoon, an artificial fresh-water lake with indented shore lines divided into three separate basins interconnected by

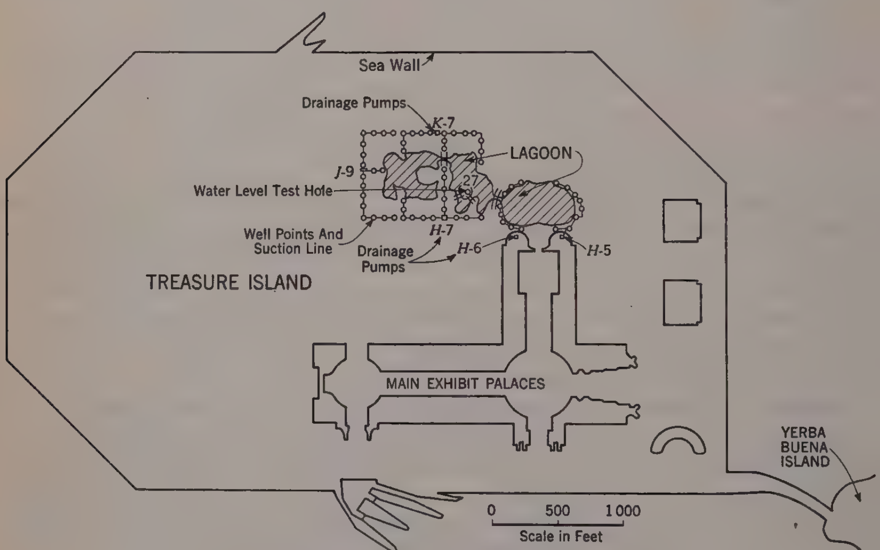

Fig. 1.—Location Map, Treasure Island Lagoon

narrow channels spanned by arched bridges (see Fig. 1). Water lillies and other aquatic vegetation grew in submerged boxes near the edges, and the adjacent margins were sloped and planted with appropriate shrubs and flowering plants.

Impermeable Lining Required

Horticultural development required that fresh water, drawn from the main water system that served the Exposition grounds, be used to fill the Lagoon. The Bay Bridge pumping plant supplied it at the average rate of 1.8 mgd

[2] "Construction Control on Hawaiian Hydraulic Fill Dam Based on Physical Chemistry," by Joel B. Cox, Assoc. M. Am. Soc. C. E., *Hydraulic Engineering*, December, 1929.

[3] "Collapsed Alexander Dam a Notable Structure," *Engineering News-Record*, April 24, 1930, p. 703.

[4] "Salt-Stabilized Road Practice Developing Rapidly," *Engineering News-Record*, July 4, 1935, p. 11.

from the San Francisco municipal system, through a 10-in. pipe line laid over the suspension spans of the San Francisco-Oakland Bay Bridge to a 3,000,000-gal reservoir on Yerba Buena Island. From that island a 20-in. pipe line across the Causeway fed the distribution network on Treasure Island.

The capacity of the Lagoon, when filled to its maximum depth of 3 ft, was 7,300,000 gal, and its superficial area was 7.62 acres. The daily evaporation rate from an open water surface on the shores of San Francisco Bay exceeds $\frac{1}{4}$ in. during the summer months. This represents a loss of 50,000 gal per day from the surface of the Lagoon.

The dredger-fill material that composes Treasure Island is largely medium to fine sand, with an occasional admixture of clay. It is quite permeable and absorbs surface water at a rate exceeding 6 in. per day. The combined daily loss by evaporation and seepage from the Lagoon, if unlined, would exceed 1,250,000 gal. As the water supply facilities were limited by the size of the bridge pipe line, it was necessary to provide an impervious lining for the Lagoon in order to include it as a landscaping feature of the Exposition.

Construction Plans

Various materials for sealing the Lagoon bottom were considered, including concrete, "bitumels," bentonite, and clay. The elements of effectiveness and economy were considered, and clay was chosen as meeting both requirements. The plans provided for excavation of three basins to a depth of 5.5 ft below the general surface, with side slopes 1 on 3. The elevation of the bottom of the excavation was + 7.5 ft above mean lower low water in San Francisco Bay. The clay seal was to have a finished thickness of 10 in. on the bottom, and for 6 ft up the side slopes a depth of 1 ft. Above El. + 10.0 ft the slopes were covered with a 2-in. minimum layer of gunite with wire mesh extending up to El. + 12.5 ft (see Figs. 2 and 3). Rock riprap was to be laid just above and below flow line at El. + 11.5 ft. The estimated quantity of material in the

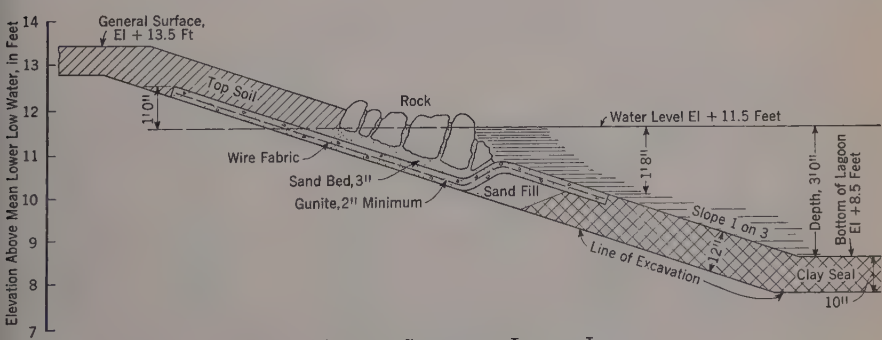

FIG. 2.—TYPICAL SECTION OF LAGOON LINING

clay lining was 11,020 cu yd compacted in place. There are 62,000 sq ft of gunite protection and 18,500 sq ft of riprap.

Selection of Clay

The specifications as prepared by the Construction Division of the Exposition Company's Department of Works required that clay for the lining should

be "a natural mixture of clay and sand occurring naturally in the borrow pit offered by the contractor," and that the character of the clay offered would be a governing factor in the award of the contract. The contractor was required

FIG. 3.—SIDE SLOPE OF LAGOON UNDER CONSTRUCTION

to submit samples of material proposed for use. It was specified that the lining material should contain not less than 35% nor more than 65% of clay, the remainder to be sand, and that it should be satisfactory for producing a watertight layer after consolidation.

TABLE 1.—MECHANICAL ANALYSES OF CLAY SAMPLES
Sieve Classification (Percentage Held)

Held by screen No.	Diameter		Sample No.:			
	In inches	In millimeters	279	280	281	288
....	0.181	4.699	....	....	....	....
8	0.093	2.362	0.10	0.33	0.88	0.58
14	0.046	1.168	0.06	1.01	1.21	0.57
28	0.0232	0.589	0.39	1.54	1.64	0.53
48	0.0116	0.295	8.24	8.46	8.37	4.08
100	0.0058	0.147	23.00	21.10	18.85	16.14
150	0.0041	0.104	4.83	3.73	4.84	4.76
200	0.0029	0.074	3.26	2.99	3.28	4.28
270	0.0021	0.053	4.17	6.37	6.26	8.38
270[a]	....	....	55.95	54.47	54.67	60.68
Total	....	....	100.00	100.00	100.00	100.00

[a] Passing Sieve No. 270.

In compliance with the specifications, samples of clay submitted from 4 borrow pits in various parts of the San Francisco Peninsula were tested in accord with the specifications. The mechanical analyses data (see Tables 1

and 2) indicated that Sample 288 conformed most closely to the size specification. Graphical plotting showed also that the material in this sample approached more nearly to the ideal grading of sizes[5] for maximum density and impermeability than any of the other samples.

TABLE 2.—SIZE CLASSIFICATION (PERCENTAGE BY WEIGHT)

Classification	Diameter, in millimeters	SAMPLE NO.: 279	280	281	288
Gravel	2+	0	0.5	0.6	0
Fine gravel	2–1	0	1.2	2.2	1.0
Coarse sand	1–0.5	2.2	2.9	4.2	1.3
Medium sand	0.5–0.25	11.8	9.6	12.7	6.7
Fine sand	0.25–0.1	23.0	22.4	16.0	18.2
Very fine sand	0.1–0.05	8.6	10.3	9.5	12.8
Silt	0.05–0.005	34.1	29.1	29.8	27.8
Clay	0.005–	20.3	24.0	25.0	32.2
Total	....	100.0	100.0	100.0	100.0

Compaction tests were run on three of the samples following the procedure outlined by R. R. Proctor,[6] M. Am. Soc. C. E., with the results shown in Table 3.

Although these tests showed that Sample 288 had slightly less density at optimum moisture content than the other samples (see Fig. 4), the difference was small and it was concluded that the greater clay content would insure a greater degree of impermeability for Sample 288 than for the others. This

TABLE 3.—SUMMARY OF COMPACTION TESTS (AT OPTIMUM MOISTURE CONTENT)

Description	SAMPLE NO.: 279	281	288
Dry weight, in pounds per cubic foot	122.5	120.0	118.0
Moisture, expressed as a percentage of dry weight	12.4	14.0	14.5
Plasticity needle reading, in pounds per square foot	1,300.0	680.0	990.0
Air voids, expressed as equivalent moisture percentages	1.7	1.2	1.4
Porosity (percentage by volume)	27.8	29.6	29.8
Allowable working moisture range [a] (percentages):			
From	12.4	14.0	14.5
To	14.9	15.3	17.0

[a] Based on a minimum plasticity needle reading of 300 lb per sq in. required to support heavy construction equipment.

sample was recommended, therefore, but it was emphasized that use of a sheepsfoot roller would be necessary in compacting it in order to secure an impermeable membrane. Arrangements were finally made with the contractor to use material from the borrow pit from which Sample 288 was obtained.

[5] "Selection of Materials for Rolled-Fill Earth Dams," by Charles H. Lee, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), Fig. 23, p. 56.

[6] "The Design and Construction of Rolled Earth Dams," by R. R. Proctor, *Engineering News-Record*, August, 31–September 7, 1933, pp. 21, 28.

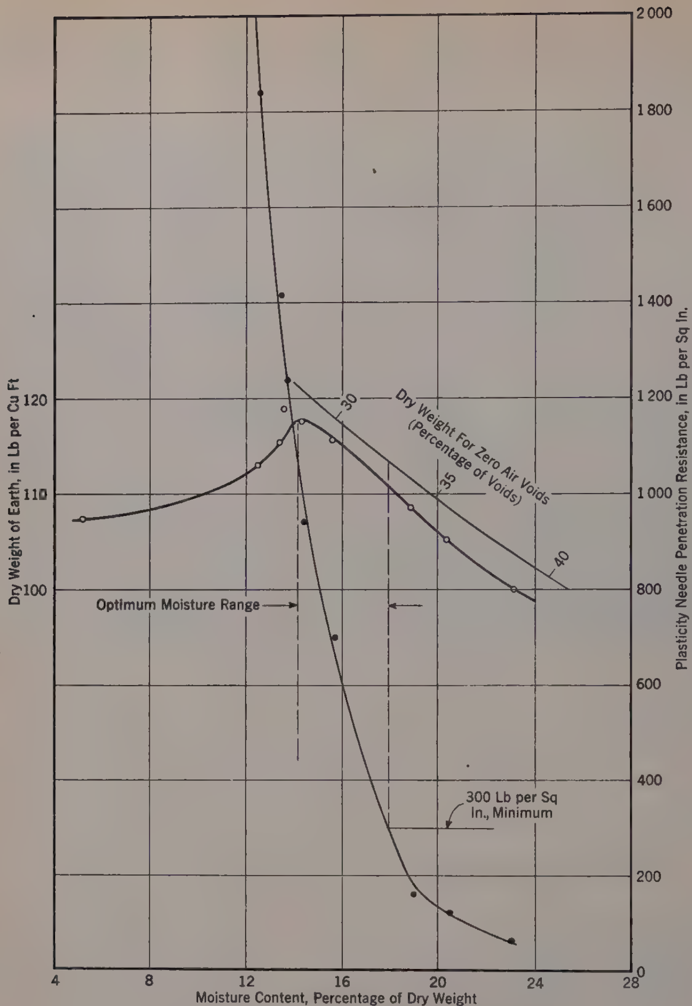

FIG. 4.—COMPACTION DIAGRAM—SAMPLE 288

TABLE 4.—CHEMICAL ANALYSES FOR EXCHANGEABLE BASES OF CLAY SAMPLE 288

Description	Exchangeable Bases					
	Calcium	Magnesium	Sodium	Potassium	Hydrogen[b]	Total capacity[c]
Exchangeable base capacity[a]	6.8	3.6	0.6	Trace	0.2	11.2
Percentage of total capacity	60.7	32.1	5.4	....	1.8	100.0

[a] Milligram equivalent per 100 g of dry sample. [b] By difference. [c] Total capacity for exchangeable bases.

The material is described as a brownish yellow clay. It is decomposed Cahil sandstone lying in place as a 10-ft layer upon the bedrock that forms a low flat-topped hill in the City of South San Francisco approximately one half mile from San Francisco Bay. The clay was in a more or less flocculated condition and could be handled without difficulty. The *pH*-value was practically neutral, tests running from 6.9 to 7.2. A chemical analysis of the soil for exchangeable bases, as made by the Division of Soil Technology, University of California, Berkeley, Calif., showed it to be a calcium clay, typical of fresh-water conditions (see Table 4).

Lining Operations

The material was excavated by power shovel after stripping the top soil, care being exercised not to go below the level of complete decomposition of the bedrock. Transportation was by barge to Treasure Island and truck to the Lagoon site. The material was spread to uniform thickness in two successive layers, each one being thoroughly rolled with a 14-ton flat road-roller. The latter was required to make a minimum of four passes and to continue until the mass was consolidated satisfactorily and until no movement of the clay appeared in front of the roller. The thickness of the first layer after compaction was 6 in. to 7 in., and the total thickness of both layers, 10 in. or more.

Material was sprinkled with water as dumped from the truck, but no effort was made to attain a condition of uniform moisture before rolling. Those in charge of construction considered that the sand subgrade had insufficient stability to support the first layer of clay under the concentrated weight of the roller, and that the feet of a sheepsfoot roller would push through into the sand and break the continuity of the clay membrane. Therefore, this type of equipment was not used.

Water was applied daily by hand sprinkling upon the finished rolled surface, both first and second layers. The first layer was exposed to the air for only a few days before spreading the second layer, but the latter was exposed nearly two months—from the middle of August until the first filling of the Lagoon on October 18. Where the bottom layer had stood for several days, systematic shrinkage cracking was evident, cracks having a depth of 2 in. or more. During the long period of exposure prior to flooding, there was more extensive cracking in the top layer. Cracks formed with typical geometric patterns, principally five-sided polygons, and in some cases attained considerable width and depth although apparently limited to the top layer.

Tests of Completed Clay Lining

Tests made under the writer's direction upon samples of the clay material immediately after rolling showed moisture content varying from 15% to 19%, and in a few cases 25%. After the lapse of one or two days, tests of rolled material showed moisture content of 13% to 16% with a hard crust.

Density tests were made also. Samples were taken from rolled material by driving, vertically, a 3-in. metal cylinder 15 in. long with sharp cutting edge to a depth of 6 in. or more, the length of the sample in place being measured by establishing a reference point outside the sampling tube and measuring from

the reference point to the top of the tube before and after driving. In this manner, error due to compaction in driving was eliminated. Volume of sample was computed from internal diameter of tube and length of sample. The complete sample was weighed before and after oven drying.

TABLE 5.—DENSITY TESTS

Sample No.	Location	Moisture content (percentage by weight)	Density; dry pounds per cubic foot
290	Near southwest corner of Lagoon 3; first layer several days after rolling	13.3	73.4
291	Near center Lagoon 3; first layer two days after rolling	15.4	69.0
297	Near northeast corner Lagoon 3; top layer at time of rolling	16.55	104.4
298	Near center Lagoon 3; top layer one day after rolling	13.74	88.7

The results of density tests shown in Table 5 indicate low density with inclusion of much air in the clay lining. The latter was due largely to the use of a flatwheel roller, which compacted a surface layer immediately beneath the wheel but did not consolidate the lower part of the layer adequately. This procedure sealed the surface and trapped the air in the lower portion of the layer, where it was held by the cohesion of the clay, probably with pressure greater than atmospheric.

Four tests made by others, two weeks after completion of the top layer, showed moisture contents of 9.9 to 11.7%, and dry weight densities of 125.7 to 133.1 lb. These samples were taken from the center of the top 4 in. and bottom 6 in. after drying and shrinkage had occurred. A 2-in. by 2-in. cylinder was used and the length of sample was obtained by measuring the final length of core cut by the tube. The dry weights as thus determined were in excess of the true values at the date of sampling due to the compacting of the sample in driving. The true values, if available, probably would have exceeded those obtained by the writer about two weeks earlier and prior to compaction by shrinkage.

Test of seepage loss through the clay lining was made September 6 to 12, 1938, about three weeks after completion, by levying off an area 45 by 60 ft and filling it to a depth of 13 in. with fresh water. The daily rate of drop in water level was uniform throughout the period of test, amounting to 1.04 in. Average daily evaporation during this period did not exceed 0.14 in., the latter being recorded at an insulated evaporating pan kept by the California Forest and Range Experiment Station, U. S. Forest Service, at the mouth of Strawberry Canyon, Berkeley, about 5 miles east of Treasure Island. Evaporation by the record is slightly in excess of that from a large water surface at Treasure Island, so that net loss due to seepage was not less than 0.90 in. per day or 187,000 gal per day over the entire Lagoon area. This was much in excess of the allowable seepage rate and it was apparent that further steps would have to be taken in order to obtain a watertight lining.

In addition to a high leakage rate, the clay was found to have softened to a mush for a depth of 8 in. or more, due to the penetration of water into the semi-pervious clay. The internal cohesion was thus reduced to the point

where trapped air could expand and escape, thus breaking and opening up the soil structure. This condition was also very unsatisfactory as it would permit the easy penetration of poles or any hard object coming in contact with the bottom.

Tests of Salt Treatment

Immediately after it was demonstrated that the membrane as rolled was not satisfactory, the writer made tests to determine the effectiveness of salt treatment in hardening the clay and rendering it impervious in the presence of fresh water. Four test samples were prepared from dry material by crushing and mixing with fresh water to a stiff plastic consistency which would hold its shape without slump, and then rolled between the hands into balls 2 in. in diameter. To the material in the first ball nothing was added, but to the other three balls salt was added, before mixing, in an amount which when dissolved in the mixing water would give the approximate salt equivalent of half-strength, full-strength, and double-strength sea water.

After drying in the laboratory for three days, the balls were in the following condition: No. 1 and No. 2 were hard and dry with light color, showing moisture film had withdrawn into ball; No. 3 was hard and partly dry with color still fairly dark; and No. 4 was firm, but very slightly plastic with dark color.

The balls were then submerged in fresh water to two thirds of their diameter. After submergence for 2 hr, the following was observed: No. 1 was very soft throughout and two thirds disintegrated with much formation of small air bubbles; No. 2 was slightly softened and about one third disintegrated below water surface with much formation of air bubbles; No. 3 showed no change above the water surface, but slight scaling and a few air bubbles were evident below the water surface, with accumulation on the bottom of the pan; and No. 4 showed no change.

After 12 hr of submergence, No. 1 was completely disintegrated, but no change was observed in No. 3 or No. 4. The same condition was found to continue indefinitely thereafter.

The rapid scaling and disintegration of the ball without salt was caused by the explosive action of entrapped air as it was being displaced by capillary water. The slower disintegration or absence of disintegration in the salt-treated balls was caused by the process of base exchange which changed the clay from a calcium to a sodium type, breaking up the crumby structure with dispersion of particles to ultimate size and formation of a sticky colloidal gel. In this condition, capillary water did not enter the ball when submerged, thus preventing softening and disintegration.

This test, when repeated in the presence of those responsible for expenditures, proved to be very convincing, and decision was made to try salt as a sealing agency.

Sea-Water Priming

Before adopting a procedure for salt treatment, consideration was given to two questions: (1) Whether sea water would be as effective as crude salt; and (2) how much sea water would be required to insure replacement of calcium by sodium in the clay lining.

A sample of sea water from the Pacific Ocean near Cliff House, San Francisco, was taken by the Division of Water Resources,[7] State of California, on August 7, 1929, and chemical analysis made, with the results shown in Table 6. This sample is representative of water in San Francisco Bay surrounding Treasure Island during the fall of the year when flow of fresh-water streams tributary to the Bay is at a minimum. The presence of appreciable calcium and magnesium was recognized as a possible source of difficulty in making rapid replacement of these elements in the soil by use of sea water. The cost of pumping sea water from the Bay was far less than the use of crude salt, and with the knowledge that clay beds washed with sea water are always in a deflocculated and impervious condition, decision was made to use sea water.

TABLE 6.—CHEMICAL[a] ANALYSIS OF SEA WATER FROM PACIFIC OCEAN NEAR CLIFF HOUSE, SAN FRANCISCO, CALIF.

Description	Na, K	Ca	Mg	SO_4	Cl	CO_3	HCO_3	Totals
Milligram equivalents per liter.....	490	21	99	52	512	0	2.6	1,151.6
Parts per million..................	11,275	426	1,212	2,500	18,200	0	159.0	33,394.0
Percentage of total................	33.3	1.3	3.6	7.4	53.7	0	0.5	99.6

[a] Na, sodium; K, potassium; Ca, calcium; Mg, magnesium; SO_4, sulfate; Cl, chloride; CO_3, carbonate; and HCO_3, bicarbonate.

Very little information was available as precedent for determining the quantity of sea water required. Tests had been made at the University of California to determine the effect upon permeability to water of a Yolo silty clay loam soil from near Davis, Calif., of applications of irrigation water possessing various compositions and degrees of purity. Water containing ions of sodium, calcium, chloride, and sulfate, in the total amount of 4,000 ppm, and in which the ratio (by chemical equivalents) of sodium to calcium was 4 to 1, was used in one such test. Surface application of this water to a 6-in. column of soil, 2 in. in diameter, in an amount corresponding to 20 times the void space of the soil, increased the soil permeability during a percolation period of 17 days from an original value of 1.2 in. per day to 7 in. per day, at which time washing by percolation was begun with salt-free water. Within 2 hr after the removal of the salt solution, the beginning of a change in the entire soil system had decreased the permeability to 1 in. per day. During the next three days the permeability had decreased still further, to 0.06 in. per day, and finally after 49 days of continuous leaching with salt-free water the low permeability of 0.02 in. per day was attained.

The ratios of chemical equivalents of replaceable ions in the soil, and of corresponding ions in the irrigation water and in the clay lining of the Lagoon and sea water are given in Table 7.

The sodium-calcium ratio of the sea water was approximately five times that of the irrigation water. In addition, the sodium-calcium ratio of absorbed bases in the clay lining was approximately twice that in the Yolo silty clay loam and the total absorptive capacity was less in the case of the clay lining. Furthermore,

[7] "Variation and Control of Salinity in Sacramento-San Joaquin Delta and Upper San Francisco Bay," Calif. Dept. of Public Works, Div. of Water Resources, *Bulletin 27*, 1931, Table 36.

the total sodium concentration in the sea water was many times greater than in the irrigation water.

An application to the Yolo soil of the irrigation water concerned corresponding in volume to 20 times the soil void space (for a void ratio of 1) represented

TABLE 7.—COMPARATIVE RATIOS OF CHEMICAL EQUIVALENTS

Material	Sodium	Magnesium	Calcium
Yolo silty clay	1	30	27
Irrigation water	4	0	1
Clay Lagoon lining	1	6	11
Sea water	23	5	1

$\frac{1{,}143 \times 62.5 \times 20 \times 0.5}{1{,}000{,}000}$ lb per cu ft = 0.71 lb of sodium per cu ft of soil, whereas the same application of sea water to the clay lining would represent $\frac{11{,}275 \text{ parts} \times 62.5 \text{ lb} \times 0.4 \times 20}{1{,}000{,}000}$ lb per cu ft = 5.60 lb of sodium per cu ft of clay. The latter application, if accumulated at the surface of the clay lining, would be 20 × 10 in. × 0.4 = 80 in. deep. In order to insure the availability of ample excess sodium for the replacement of calcium in the clay, plus an

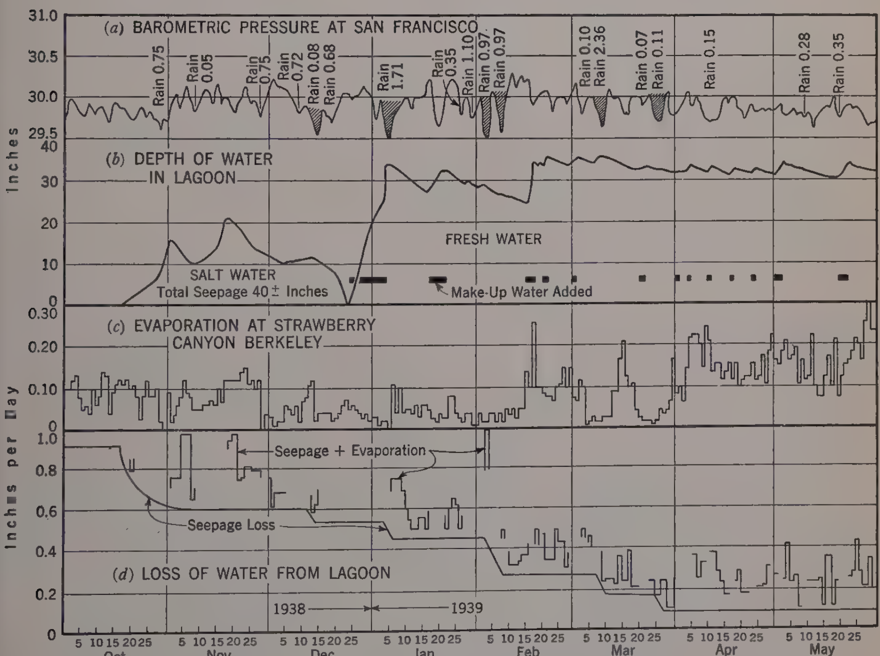

FIG. 5.—SEEPAGE LOSS FROM LAGOON AT TREASURE ISLAND

excess of water for waste by evaporation, leakage other than seepage, etc., it was decided to pump sea water into the Lagoon until a total depth of 40 in. had escaped. If all sodium in this water were absorbed by the clay, it would repre-

sent 2.80 lb of sodium per cu ft of clay, or four times that available in the laboratory experiment.

Although this quantity of sea water included a generous margin, it could be provided with little additional cost, since power was being purchased under a general wholesale rate for all Exposition uses and pumping equipment was already on hand.

Sea water from San Francisco Bay was pumped into the finished Lagoon beginning October 20, 1938, and continued until a depth of 18 in. was attained which completely submerged the clay membrane. This was found to produce an immediate reduction in seepage from the rate of 0.90 in. per day with fresh water to 0.60 in. (In Fig. 5, the seepage-loss line is determined by subtracting the observed evaporation at Strawberry Canyon from the loss of water from the Lagoon for periods of four or more days.) Furthermore, the clay, although softening to a depth of 2 in., was hard and cohesive for the remainder of its depth. In order to impregnate the clay lining thoroughly with salt water, pumping was continued until December 24, 1938, when the accumulated depth of seepage had reached a total of 40 in. At this time all remaining salt water was drained from the Lagoon and fresh water run in to a depth of 34 in. (see Fig. 5).

In the interval prior to filling with fresh water, samples were taken for physical and chemical tests, and it was found that the consistency below the 2-in. depth was still cohesive and hard. The results of laboratory tests are given in Table 8.

TABLE 8.—Chemical and Physical Data for Samples of Clay from Lagoon Lining After Priming with Salt Water

Exchangeable bases	Exchangeable Base Capacity Milligram Equivalent per 100 g of Dry Sample			Percentage of Total Capacity		
	Sample 309 top	Sample 310 center	Sample 311 bottom	Sample 309	Sample 310	Sample 311
Calcium	2.6	2.8	1.8	18	14.5	17
Magnesium	7.6	8.7	4.3	52	45.5	41
Sodium	4.4	7.6	4.4	30	40.0	42
Potassium	Trace	Trace	Trace	Trace	Trace	Trace
Hydrogen	....	....	....	....	....	....
Total capacity for exchangeable bases	14.6	19.1	10.5	100	100	100

The moisture equivalents applied to these tests were as follows:

Sample	Moisture equivalent (percentage of dry weight)
309	23.8
310	26.0
311	19.5

Comparison of the ratio of replaceable sodium to calcium in the natural and salt-treated clay showed a change from 1 : 11 in the original Sample 288, to 1.7 : 1 in Sample 309, 2.6 : 1 in Sample 310, and 2.4 : 1 in Sample 311. This

represented a 24-fold increase of sodium over calcium with only a 30% increase in the total replaceable bases. The moisture equivalent tests indicated a material highly retentive of moisture. The tests were thus indicative of extensive replacement of calcium by sodium and gave confidence as to the ultimate reduction of seepage when deflocculation was made possible by the removal of excess sodium in the soil solution by leaching with fresh water.

Fresh-Water Filling

Soon after fresh-water filling was begun, it was noted that great quantities of air were held in the upper section of the clay bottom of the Lagoon. This could be released by dragging the end of a rod along the bottom. It came to the surface principally in clouds of small bubbles, but sometimes as large bubbles. No air was noted escaping except when the bottom was disturbed.

The seepage rate during the first month after filling was constant at 0.46 in. per day (see Fig. 5). Suddenly early in the second month, without apparent reason, the rate dropped to 0.28 in. per day and continued thus to the early part of the third month, when a further drop to 0.18 in. per day occurred. Later in the month this reduced to 0.10 in. per day. Upon investigation of the continuous record of barometric pressure at the San Francisco station of the U. S. Weather Bureau, it was discovered that unusually low barometric pressures were recorded just preceding and during each of these periods of decreasing rate (see Fig. 5). It was also noted that after the second low barometric period, the quantity of air held in the top layer of clay was noticeably less

The conclusion is believed to be justified that low barometric pressure in the surrounding atmosphere produced an unbalanced pressure in the clay voids containing trapped air, which was great enough to overcome cohesion and permit release of the air. Such release would suddenly rearrange the particle structure and permit fresh water to enter and wash the walls of voids previously filled with air. This would allow more complete dispersion of colloidal particles and the filling of the voids with sticky colloidal gel. The clay lining thus would be rendered more impervious to water.

The final seepage rate of 0.10 in. per day is within the allowable limit and permitted replenishment of losses from the Lagoon without excessive draft on the Exposition water supply.

Ground-Water Behavior

In connection with seepage, a study was made of the behavior of the plane of saturation in the fill beneath the Lagoon. For this purpose water levels were observed at intervals of a few days in a 3-in. auger test hole lined with perforated casing and located on a sand island which was built on top of the completed clay lining (see Fig. 1). This test hole (No. 27, Fig. 1) penetrated the clay layer and extended beneath it a distance of 10 ft with a total depth of 15 ft. The record of water level in test hole No. 27 is reproduced graphically in Fig. 6, together with relative elevations of clay lining and water surface in the Lagoon. This record shows clearly the combined effects of increased depths of water in the Lagoon and of decreasing permeability of the clay lining.

Continuous drainage of Treasure Island was necessary to prevent rise of ground water to the level of ornamental tree roots. Seepage from the Lagoon was anticipated as one of the important sources of ground water, and in order to prevent excessive local accumulation five drainage pumping units were installed near the Lagoon with well points completely encircling it at intervals of 100 ft (see Fig. 1). By this means the ground-water increment from the Lagoon is drained away, and the water level in the fill surrounding the Lagoon is maintained between El. − 1.0 ft and El. − 2.0 ft.

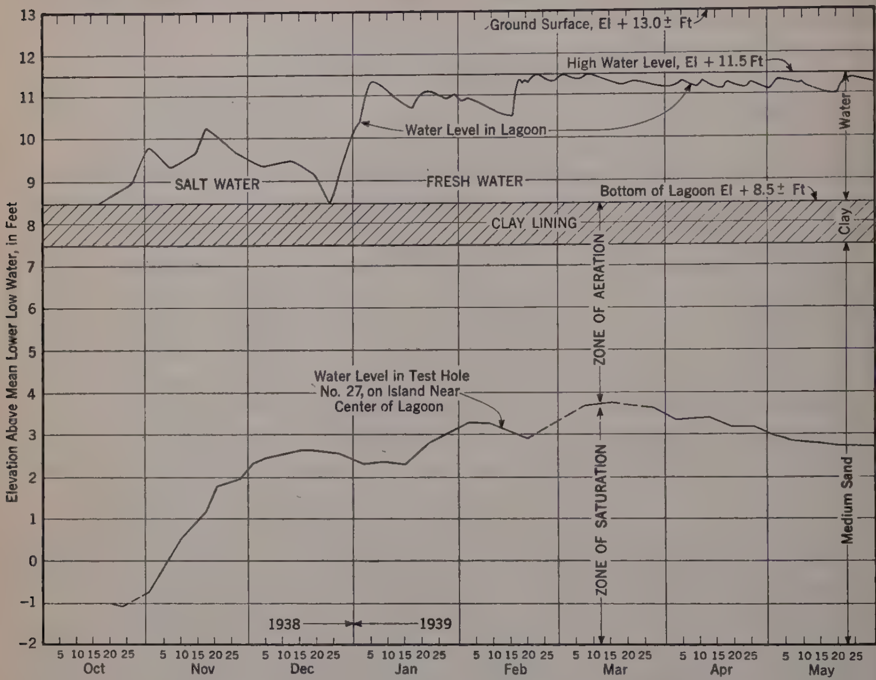

FIG. 6.—RELATIVE ELEVATIONS OF WATER LEVEL IN LAGOON AND IN UNDERLYING SAND FILL

The close relation between seepage and pumping is illustrated by comparing the increased rates of pumping and seepage after the Lagoon was filled. During the eleven weeks prior to filling with salt water (October 18, 1938) the total pumpage from all five drainage units was 4.44 acre-ft per week. The approximate average rate of pumping during the eleven weeks after filling was 7.21 acre-ft per week, an increase of 2.77 acre-ft per week. The rate of seepage during this period (see Fig. 5), if applied to this entire Lagoon area, was equivalent to 2.45 acre-ft per week. As pump records were probably in excess of actual quantities pumped, these values can be considered as in close agreement.

The rise in water level in test hole No. 27 represents the accumulating ground storage under the Lagoon required to produce sufficient gradient for water to flow to the well points. During the first three weeks after filling, the rate of rise was 1 ft per week, which, if it were uniform over the entire area, would

represent 1.8 acre-ft per week, using the drainage factor of 0.23 as determined by large-scale field test. Subsequent rate of rise up to the middle of March represented 0.2 acre-ft per week. After that date the water level fell, indicating that seepage was becoming insufficient, even with full Lagoon, to maintain the initially created gradient of the water table.

Effect of Salt Treatment Upon Quality of Water

Tests made upon samples of water, taken from the Lagoon 4.5 months after filling with fresh water, showed no appreciable increase in mineral content over that of the source of supply. The *pH*-value was practically neutral, ranging from 6.9 to 7.2. Aquatic vegetation grew normally, with roots submerged in the water. Apparently all excess mineral salts, remaining in the soil solution of the clay lining after drawing off of the salt water, were leached out by the fresh water and were carried on down into the underlying sand. As soon as the soil solution was freed of unattached sodium ions by leaching, the clay particles were dispersed, forming a sticky gel that filled all voids. In this form the sodium remains fixed as long as it is in contact with fresh water. The quality of fresh water in the Lagoon, therefore, was unaffected by the salt treatment.

Conclusions

The clay lining of Treasure Island Lagoon was sealed successfully by salt-water treatment at a cost of a few hundred dollars, without interfering with other necessary operations. To have sealed it by other methods, such as application of bentonite, would have cost at least $15,000 and would have involved delay in completing the landscaping features.

In view of the fact that fresh water used in filling the Lagoon subsequent to the salt-water treatment did not experience an increase in salinity, this method of sealing would appear to have a wide field of usefulness in constructing impervious membranes for water works structures such as reservoirs and dams, as well as those for recreational and other purposes not involving potability of water.

Acknowledgments

Treasure Island Lagoon was designed and built by the Department of Works, Golden Gate International Exposition, of which W. P. Day, M. Am. Soc. C. E., was director. The Division of Soil Technology, College of Agriculture, University of California, made chemical tests of the clay used in the lining. G. B. Bodman, of this Division, confirmed the writer's conclusions as to efficacy of salt-water treatment, and advised regarding the quantity and methods of application.

Evaporation data were furnished by E. I. Kotok, director, California Forest and Range Experiment Station, U. S. Forest Service, and continuous barometric pressure records and rainfall data were furnished by Edward H. Bowie, meteorologist, U. S. Weather Bureau. Soil testing for the selection of material for the clay lining and experimental work leading to the adoption of salt-water treatment were conducted at Pacific Hydrologic Laboratory under the writer's direction.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

FLOOD-CONTROL METHODS: THEIR PHYSICAL AND ECONOMIC LIMITATIONS

PROGRESS REPORT OF COMMITTEE OF THE HYDRAULICS DIVISION ON FLOOD CONTROL

Foreword

1. Realizing that the public at large and many engineers as well do not have a clear conception of the limit and extent to which floods can be controlled, nor of the economic problems involved, the Board of Direction of the American Society of Civil Engineers, at its meeting in January, 1937, authorized the President of the Society to appoint a Committee on Flood Control, to make a general appraisal of flood-control methods, with particular reference to their physical and economic limitations. This report embodies the findings of the Committee so appointed.

Introduction

2. There is a growing tendency on the part of the general public to regard flood control as a remedy of ready application, capable of affording a dependable cure for the flood hazards that exist throughout the United States. This belief is not justified. Apparently, there is a widespread belief that floods are greater and more frequent than they used to be, due to the works of man. Moreover, it seems to be the impression that it is feasible to prevent floods by holding the water back upon the headwaters of the streams. These impressions have been intensified by the increased publicity nowadays given to great floods and other catastrophes by means of the radio and the moving picture. These mediums of publicity create sympathy for the suffering caused by floods and a desire to change the conditions which foster floods. In so far as these emotions upon the part of the public can be directed toward worth-while relief projects that are feasible of accomplishment, they tend to promote the public welfare. It would be unfortunate, however, if the present state of mind should lead to unwise expenditures, with results detrimental to future flood-relief undertakings. Therefore the present trends of thought need to be clarified, with special reference to the questions of (*a*) what flood control may be expected to accomplish; (*b*) what are the means of bringing it about; and (*c*) what are the relations between costs of flood control and benefits to be derived.

3. There are many methods for securing flood protection, flood reduction, or general relief from the effects of floods, as given in this report. Nearly all these remedies have some useful application but all of them have their limita-

tions. Moreover, the widely used term, "flood control," itself may convey to the public an erroneous impression of the actual results to be obtained. Floods cannot be stopped; they should only be controlled to such an extent as may be warranted by benefits.

4. The problem is difficult on account of its size and the multitude of facts that must be considered in each case. It is affected by the great area that must be surveyed, by the number of remedies that are adaptable only to certain cases, by the fact that relief is often very costly in proportion to benefits, and by the infrequency of great floods. It is complicated further by the necessity for correlating local solutions throughout river systems, so that one shall not interfere with another.

5. The various means of flood control known to man have been practiced for centuries and all of them date back to the earliest civilizations. All possess decided limitations and none furnishes a panacea for flood mitigation in general. The popular belief that flood control is a definite public works problem, in the same category with grade-crossing elimination on the highways, is a fallacy. Flood control is fraught with many uncertainties, and becomes even more uncertain when linked with various forms of water utilization. Hereinafter the means for flood relief and their limitations are discussed: First stating certain definitions; then the question of the "provisional flood" to be used as a measure in the comparison of various remedies; then the remedies—storage, channel improvement, and other means of relief; and finally, certain aspects of the problem, including scope of engineering studies, costs and benefits, and the agencies through which flood relief may be secured.

Definitions

6. Flood control concerns the lands bordering on rivers, and consists in protecting areas from overflow in one or more of the following ways: By means of levees and walls; by improving the discharge capacity of river channels; by reducing flood crests through detention of part of the flow in reservoirs; and by diverting excess waters into by-passes or floodways. The degree of control so effected may vary widely, depending upon: (*a*) Limitations imposed by physical aspects; (*b*) economic justification, in view of benefits to be expected; and (*c*) availability of funds. According to the degree of protection afforded, flood control may be either partial or complete.

7. Partial flood control consists in providing control works designed to give protection against floods of lesser magnitude but of greater frequency, with no attempt to protect against great floods of rare occurrence. Most flood-control works built to date in the world at large are of this type.

8. Complete flood control consists in providing works designed to protect against floods of all magnitudes, including the greatest estimated as physically possible of occurrence. This type of control is seldom attainable for large areas and, up to the present time, has been applied successfully only to municipal or industrial areas, and in some cases to other areas in which property values and capital investment are sufficiently large to justify the cost involved.

9. Watershed protection consists in preserving the vegetal cover; in providing numerous small reservoirs on headwater streams; in gully control by

means of check dams; and in proper farming methods, essential for preventing the soil from being washed from the land and choking river channels. All watershed protective measures are valuable as adjuncts to flood control but in themselves do not constitute flood control, though they are often mistaken for it.

Provisional Floods

10. In considering a particular case of flood relief, a wise decision requires comparison of costs and benefits, as well as comparison of alternative methods of relief. Obviously comparisons must be upon the same basis. A so-called "provisional flood," therefore, should be selected as a basis for planning adequate relief measures and estimating their costs. An actual great flood is often convenient for this purpose. In the protection of the high-value portions of cities by means of levees or dams and in cases involving the possibility of great loss of life, a substantial factor of safety should be used. In the protection of agricultural lands, and in some other cases, it may be advisable to adopt a factor less than unity; in such cases protection against moderate floods may be economically sound, whereas complete protection may be economically impossible.

11. In general, the provisional flood should be the greatest flood for which reasonably dependable figures can be made available. Consideration should be given to river-valley storage, before and after the proposed works are built, where such storage would be important, as well as to the probable duration and character of the flood itself and to the effect of silt and debris.

12. Flood Records.—The period of years covered by systematic stream gaging in the United States is so short as to render it unwise to base the probable maximum flood in a given watershed upon the records of a single stream. Valuable information relative to floods can be obtained sometimes from an investigation of the fragmentary records of the distant past. For example, the flood of 1936 at Pittsburgh, Pa., was 7.3 ft higher than any flood since 1855, the beginning of systematic records at Pittsburgh, but only 4.8 ft higher than any flood since 1763. The necessity for keeping flow records on the principal streams is emphasized by the immense importance of such records in designing works for flood reduction. This class of information can be obtained only by years of careful and continuous observation. It is manifestly the duty of governmental bodies to maintain this hydrographic work consistently. The subject of flood records has been under investigation for several years by the Society's Committee on Flood Protection Data.[1]

13. The many factors that influence rates of flood flow on different watersheds combine to produce wide variations in the runoff per unit of area. Recorded peak flood rates per square mile on streams of the United States range from 2 to more than 3,000 cu ft per sec.

14. The predominating flood influence is precipitation in the form of rain. The amount of rain with reference to time, the seasonal distribution of rain and the periods of intensity are of especial importance in the creation of floods. In the northern latitudes, the accumulated precipitation as snow is important.

[1] *Proceedings*, Am. Soc. C. E., March, 1935, p. 333; February, 1936, p. 203; March, 1937, p. 539; February, 1938, p. 333; and January, 1939, p. 93.

The most intense rainfall rates occur in small areas, the rate diminishing materially as the area increases. This fact largely accounts for the great differences in the unit flood rates of large and small watersheds.

15. An important influence in creating great floods is exerted by the characteristics of the watershed, including its size, its shape, the arrangement of tributaries, the topography and geology, the permeability of the surface, the vegetation growing thereon, and the storage capacity of the subsoil. The climate of the region affects rates of delivery of precipitation to the stream, not only through precipitation as snow, its accumulation and melting, but also through the effect of climate upon the imperviousness of the ground and the character of the vegetation.

16. Seasonal Variation and Frequency.—Where it can be shown that flood reservoirs are adaptable to other useful purposes, such as water conservation or power, seasonal flood peaks become significant. Other things being equal, the barren or leafless season of the year produces the greatest flood peaks. On the other hand, on the smaller streams the retarding effect of summer vegetation may be more than offset by more intensive summer rains, and small watersheds, therefore, may produce the greatest flood peaks in the warm season of the year. On many rivers disastrous floods may occur in any month of the year.

17. The probable frequency of destructive floods has an important bearing upon justifiable expenditures for flood relief. The records available on any particular river at the present time are insufficient to indicate the probabilities of the future, except as regards the frequency of the smaller floods. In the light of many great floods in the recent past, it is advisable to revise earlier studies of magnitude and frequency in the various parts of the United States. For some rivers such revisions have already been made.

18. Comparison with Other Watersheds.—Provisional floods should not be selected without a comparison with great floods upon other streams. Peak rates of flood flow per unit of area are affected by so many factors that the comparison of one watershed with another becomes very difficult. Several formulas and methods of procedure have been suggested to facilitate such comparisons, which are necessarily based upon occurrences of the past. Although, in the United States, no record of the flow of a particular stream covers a long enough period to determine by itself a probable maximum, it is not unlikely that a maximum flow occurs on some watershed nearly every year. It becomes important, therefore—difficult though the task may be—to search the data of other streams for guidance in selecting a provisional flood.

19. New methods of weather forecasting by air-mass analysis[2] give promise of enabling meteorologists to estimate the maximum rainfall to which a locality may be subject. Application of appropriate runoff relations, if such could be determined, would give the maximum flood which might occur. However, a word of caution in using this method is offered, since the difficulties of accurately estimating rainfall rates and areas covered, coefficients of imperviousness, natural storage, and resulting runoff even for small areas, are well known, and

[2] See Progress Report of the Committee on Flood Protection Data, *Proceedings*, Am. Soc. C. E., January, 1939, p. 93.

for large areas these difficulties are greatly increased. At the present time extensive studies are under way to work out these relations, and knowledge and data are increasing rapidly. It is hoped that methods will be evolved worthy of general use.

20. California Conditions.—One of the regions which suffers severely from floods is California. A high mountain range relatively close to the sea, a narrow coastal plain, and a heavy concentration of population on this plain in the southern part of the state have produced a flood problem of great complexity, differing materially from that obtaining east of the Rocky Mountains. The menace of high flood flows is greatly increased by immense quantities of sediment and detritus accumulated in the descent of the flood waters from the mountains, by the passage of this material through the foothills, and by its deposit upon the thickly inhabited, flat, coastal plains. Another area noted for its extraordinary flood intensities is southern Texas.

21. On account of heavy flood damage in past years and the rapidly increasing population of the Los Angeles region, large expenditures have already been made for flood relief. The unprecedented flood of March 2, 1938, at Los Angeles, Calif., covering about 2,500 sq miles, caused the loss of 113 lives, and property damage of $45,000,000.

22. Cloudbursts.—The runoff from severe cloudbursts in the many thousands of smaller streams in the United States is said to cause an average annual damage exceeding that inflicted by floods in the principal rivers. Some cities have protected themselves, at great cost, against this type of flood. The fact that such floods descend from any terrain so situated as to promote the air-current conditions requisite for producing cloudbursts renders the problem extremely difficult to solve. This applies in particular to railroad and highway systems, for which culverts and drainage facilities are rarely designed with capacity sufficient to pass cloudburst runoff, and which, therefore, remain subject to washouts.

23. Ice Jams.—The rivers of the northern United States occasionally subject the adjacent lands to floods of considerable depths on account of ice jams. On some of the rivers in that section, the high-water marks produced by ice-jam floods are as high as, or even higher than, those produced by the greatest floods unaffected by ice. The ice-jam flood is largely unpredictable both as to time and place of occurrence. Where relief measures are planned, the presence of ice and trash should be guarded against at places where the flow of flood waters is restricted, as in the case of detention basins. Ordinary works for flood relief provide little relief from ice jams.

Storage as a Remedy For Floods

24. Two general types of water-storage structures have been utilized for flood-reduction purposes—namely, storage reservoirs and retarding basins. Until recently, the use of storage reservoirs as a flood remedy has been quite limited in the United States. The retarding basins built by the Miami Conservancy District in Ohio, in use for 15 to 20 years, are capable of storing about 841,000 acre-ft, and comprise one of the outstanding projects of their kind, designed exclusively for flood control.

25. The Corps of Engineers, U. S. Army, has studied the problem of reducing the floods of the Ohio River and its principal tributaries and has reported upon a comprehensive plan for flood control by reservoirs. Several of these reservoirs are under construction in Ohio and Pennsylvania, and fourteen more have been authorized. Many of these reservoirs are intended to serve the multiple purposes of flood reduction, water power and water storage. The report[3] concludes that a feasible number of reservoirs would be insufficient for protection upon the Ohio River proper and that extensive levees and flood walls would be required. Several great reservoirs in the southwest have been built for the double purpose of flood reduction and conservation. These include the Elephant Butte Dam on the Rio Grande and the Hoover Dam on the Colorado River. Other large projects of this type are being planned or constructed.

26. Multiple-Purpose Reservoirs.—Multiple-purpose reservoirs, to be useful when needed for flood relief, must at that time be empty to the extent counted upon for flood protection and after use must be emptied again promptly to be reserved for the flood which may follow immediately. This part of the reservoir, therefore, must not be used for any other purpose. It is believed that the public has been seriously misled in many cases to believe that, by providing power facilities or other benefits, flood control has been secured at a nominal cost.

27. Stored flood water has a definite cash value if there are facilities to use it for power or irrigation. The release of flood waters, once they have been stored in reservoirs, is contrary to the modern concept of water utilization and conservation. However, failure to release such water promptly tends to defeat the purposes of flood control and renders flood-control reservoirs liabilities, rather than assets, to the property owners who depend upon them for protection.

28. Flood Reduction by Storage.—It is characteristic of great floods that the river stage rises more or less rapidly to a comparatively sharp peak, subsiding somewhat more gradually as the water is drained away after heavy precipitation ceases. Destructive peak rates may continue for a period of a few hours on a drainage area of 10 sq miles up to a few days on 1,000 sq miles and as much as several weeks on 100,000 sq miles.

29. Flood reduction by water storage involves diminishing the flood flowage upon certain lands by increasing the flood flowage upon the lands immediately above the storage dam. No protection whatever is afforded to that part of the drainage area above the storage dam. This procedure is economically feasible only when the lands to be protected are more valuable than the lands upon which the flowage is increased. This condition sometimes obtains when the overflowed lands above the dam site can be used to store flood waters of considerable depth. The lands or property to be protected also must have sufficient value to warrant the cost of the reservoir construction. The lands included in the sites for the great reservoirs in the arid section are very often publicly owned, desert or forest areas, over which the federal government permits easements for reservoir sites without cost.

30. Generally speaking, storage reservoirs for flood reduction alone are not practicable in either relatively flat or excessively steep country. In fact, unless

[3] H. R. Doc. No. 306, 74th Cong., 1st Session.

topographical conditions are especially favorable for dam sites with ample storage capacity this type of flood relief is not economically feasible. Where flood prevention and water conservation can be accomplished jointly, however, the construction of reservoirs in flat country may be justified.

31. Outlets and Spillways.—The type of outlet to be used in a particular case is dependent upon rates of flow to be controlled, depth of water, other incidental purposes for which the water is stored, local conditions, and the policy to be adopted in the operation of the works. In the early flood-control reservoirs built in the United States the outlets were usually of the uncontrolled type, but in recent works the tendency is toward controlled outlets. In reservoirs designed for more than one purpose the outlets are nearly all of the controlled type.

32. The distinguishing feature of a retarding basin is its uncontrolled outlet, which renders its operation automatic. Experience in the United States and abroad indicates that the retarding-basin principle is well adapted to small watersheds. On the other hand, if outlets are controlled, the outflow can be regulated in such a way as to keep pace with decreasing basin capacity caused by sedimentation. Furthermore, setting a gate at a fixed opening permits of utilizing the retarding principle effectively, whenever this mode of operation appears advantageous during a part of the year. In the southwest many outlets have been put out of service due to drift and silt during floods.

33. Ample spillways are required, in order to assure the safety of earth dams, the failure of which might be far more disastrous than the floods against which they are designed to furnish protection. In some cases, such a spillway with a large factor of safety can be built at comparatively little extra cost over one with a small factor, by utilizing adjacent topography to the best advantage. In the Miami dams a flow 100% greater than that of the 1913 flood, an epochal flood, could be accommodated without structural injury to the dams.

34. Reservoir Lands.—The studies of flood control for central Ohio indicate that, in the case of reservoirs designed to accommodate very great floods, the maximum area of reservoir-bottom lands subject to frequent flooding would not exceed 20% of the flooded area with the reservoirs filled. An additional 40% would be completely covered only about once in 100 years, according to past experience, and the remaining 40% would never be flooded, if the experience of the previous century were typical of the future. In the case of the retarding basins of the Miami Conservancy District, the flowage land was purchased and then resold or leased, except in the lower parts of the basins which are subject to frequent flood hazard, thus reducing the net cost of land for the improvement. This is possible only where flood and climatic conditions are favorable.

35. Drift and Sediment.—One of the lasting impressions created in viewing the effects of a great flood is the quantity of drift carried by the flood waters. East of the Rocky Mountains, the drift often can be controlled in the flood reservoir by suitable barriers which become effective when a comparatively small amount of storage depth has been attained.

36. In many sections of the United States, the problem of detritus and sediment is of major importance, tending to shorten the useful life of reservoirs, contributing in marked degree to the cost of flood relief and in many cases determining the measures to be adopted. In some instances, particularly where there is heavy flow of water through a comparatively small reservoir, sufficient sediment may be deposited to fill the reservoir in a few years. In estimating the useful life of reservoirs this effect should be duly considered.

37. Operation of Storage Reservoirs.—Where the construction of multiple-purpose reservoirs is warranted, all purposes can be accomplished in one capacious reservoir with safety, provided that in operating the works space is always reserved for the storage of flood waters in seasons when excessive floods may occur. No great harm is likely to be done if water power and water conservation encroach upon each other to some extent. It is vital, however, that adequate space be reserved strictly for flood relief. If suitable reservoirs are skilfully operated, they can be made to furnish a high degree of protection to the valley lands immediately below them.

38. Up to the present time storage reservoirs for flood relief have been built and operated by the local agency most directly concerned. The work of this character already completed, under way, and projected is such that serious consideration must be given to the coordination of flood-relief works, in order that they may operate most effectively and, in fact, without actual harm. If reservoirs are to be gate-controlled, an agency capable of making wise decisions promptly in an emergency should determine how to control them. Such decisions would include, for example, whether the greatest good will come from restraining the flow, thus protecting certain communities, or in releasing flow, in order to reserve space in the reservoirs for possible greater inflows. The coordinated use of flood-control projects now in prospect on the Ohio watershed, for example, is believed to require a strong central authority, provided with a trained personnel, permanently engaged on this problem so as to acquire a cumulative experience, equipped with ample facilities to ascertain rapidly changing conditions, and free from local influences.

39. Limitations of Storage.—The beneficial effects of storage for flood reduction grow progressively less from point to point downstream below the storage structure, due to the contributions of uncontrolled tributaries and the return flow from valley storage. Consequently, the problem of effectively operating reservoirs located on the headwaters in such a way as to save property along the main stream is a serious one.

40. Next to operating a reservoir to the best possible advantage for reducing a flood crest, the most important problem confronting the operator is the release of the stored waters. As flood-storage capacity in a reservoir requires emptying promptly after each filling, in order to make this capacity available for the succeeding flood, the release of the impounded waters, especially from a group of reservoirs in the same watershed or neighboring watersheds, may result in building up new flood peaks in the main stream, especially if such outflows should synchronize with flood waters from uncontrolled tributaries. Operating difficulties in this respect are especially critical in watersheds where great storms are known to occur in quick succession. A large part of the

United States is subject to recurrent storms, producing what is known as "twin floods," with peaks often only about 8 days apart. If, during the interval, bank-full stages prevail in the river below the reservoirs, the release of stored water will necessarily cause overflow of lands intended to be protected. Delaying the release may invite a still greater overflow, should the next flood crest go over the spillway. Reservoirs of enormous capacity, capable of holding all excess waters in such emergencies, therefore, are desirable; but natural sites for such large reservoirs are difficult to find. The problem is complicated further because errors in operation may affect riparian rights and navigation interests adversely.

41. In the case of a system of retarding basins, the chief disadvantage is the inability to exercise full control in emergencies and to regulate the discharge in such a way as to avoid building up new flood crests at points downstream. A major objection to the usual type of retarding-basin outlet is the impossibility of removing deposits of sediment from the river channel below such basins by systematic flushing.

42. Although during moderate floods small dams function well in reducing runoff and promoting infiltration, during major floods the existence of such dams may become responsible for increased river stages and the flooding of lands which otherwise would not be inundated. Failure of small dams has been known to occur during great floods, releasing additional water at a critical time.

43. The beaver dam is a type of small dam being advocated in some states for flood control. These dams are not well constructed, frequently go out in flood, and therefore constitute an actual flood menace.

44. Unless reservoirs are planned and operated with due regard to channel maintenance in the rivers below them, there is danger of deterioration of river channels in the use of storage projects to reduce floods. There are two forms of channel deterioration: The loss of carrying capacity for both water and solids, caused by elimination of the flushing action of bankfill river stages; and excessive erosion of river beds immediately below dams, due to insufficient loads of sediment in the waters released from storage, a condition which causes the kinetic energy normally expended in debris transportation to pick up the river beds. Both forms of deterioration are apt to develop in different parts of the same stream and produce deleterious changes in hydraulic gradients. Similar conditions in European streams have necessitated large expenditures for sill dams, to hold river beds in place, and for protection of foundations of bridge piers and other structures.

Channel Improvement as a Remedy for Floods

45. The improvement of the channel of a stream to increase its carrying capacity is probably the oldest and most frequently used method of securing relief from overflows. The three common methods of channel improvement are channel enlargement, levee construction, and the shortening of channels by means of cutoffs.

46. Channel Enlargement.—The enlargement of the channel of a large river by widening or deepening is seldom a practicable method of flood relief, except

for short distances in the protection of cities. In the case of comparatively small streams passing through cities, however, this method of relief is commonly employed.

47. Such improvements are frequently accomplished by lining the bottom and sides of the channel to prevent erosion and to increase capacity. Where the population is dense and adjacent property is sufficiently valuable, it is common to cover such channels and utilize the top for streets or parking areas, or to surmount the cover with suitable fill and create small parks and playgrounds. In a few instances, the construction of masonry retaining walls for short distances, bordering valuable urban property, has been justified, in order to secure adequate flood cross section with a minimum of land taken.

48. There may be danger, however, in channel enlargements made locally, due to the production of unstable channel conditions above and below the improved reach. In general, river improvement of this type must be carried on over considerable distances in order to produce acceptable hydraulic conditions and avoid injury to riparian interests or high-maintenance costs.

49. Levees.—For the purpose of flood protection, levees have been extensively used in the United States and elsewhere. Their chief merit is small cost of construction and maintenance. Lands adjacent to the Mississippi River have been protected by levees for more than two centuries, and the practice has been extended to many other parts of the United States where the increased value of the crops protected from flood damage has been sufficient to warrant the construction costs involved. Usually it is not feasible economically to guard against the greatest flood that may occur.

50. In comparison with natural conditions, levees, through confinement of the flood flow, increase flood stages, decrease valley storage, and cause increased velocity of flow in the channel. On the lower Illinois River, for instance, for a distance of 90 miles, 14 levee districts have reduced the width of the flood plain about 90% and have changed the cross-sectional area of the flowing stream in a great flood to some 25% of that available prior to construction of the levees. The result has been an increase in the flood height at the head of the levee system of not less than 5 ft, together with serious losses due to breaching of levees.

51. These structures, as a rule, are subject to seepage, slides, and subsidence, because of necessity they are built of materials procured locally, and because poor foundation conditions are often unavoidable. They are also damaged sometimes by rodents. Moreover, if a breach is made in a levee system consisting of a long, single line of embankment, large areas of land are likely to be inundated. Finally, levees in the northern part of the United States may be subject to damage by ice.

52. Valley Storage.—Any project to exclude floods from agricultural lands necessarily robs the valley of natural storage and therefore tends to increase rates of flow downstream. Studies of the Miami Conservancy District showed that the maximum flow at Hamilton, Ohio, under the improved channel conditions, would be increased about 40% compared to the actual flow of the 1913 flood, on the assumption that the lands which had been flooded above Hamilton would be protected by levees. A similar study of increased flow at

Dayton, Ohio, due to the elimination of valley storage above the city, indicated an increase of 35%.

53. Cutoffs.—One of the methods of increasing channel capacity or reducing flood stages is to shorten the distance traveled by the water, thus increasing the slope and the resulting velocity by the use of the cutoff. It is characteristic of rivers in alluvial valleys that sometimes the sinuosity becomes very great. This is particularly noticeable on the Mississippi and Missouri rivers. From time to time a cutoff occurs naturally by breaking through the neck of a loop. The artificial creation of cutoffs by dredging is a common feature of reclamation of agricultural lands. Recently, cutoffs have been widely utilized on the lower Mississippi River by dredging relatively small channels across the necks of loops. Concentrated fall and increased velocity cause the cutoff to enlarge greatly in successive floods.

54. The history of cutoffs reveals favorable as well as unfavorable consequences and invites clear understanding of the limitations involved. A common practice in Europe has been to build a completely new river channel in the dry, open this new channel and simultaneously close off the old channel, leaving the adjustment of river bed and hydraulic gradients to work itself out. In many cases this plan has resulted in damage to river channel and to riparian property above, as well as below, the cutoff. It produces a steep slope in the new channel, with attendant scour and redeposition of the scoured materials at points below. The benefits resulting from cutoffs, therefore, often may be offset by the detrimental effects mentioned. In addition, on navigable rivers cutoffs have been known to reduce depths to less than project depth at low stage and create velocities detrimental to navigation. Moreover, in rivers carrying appreciable sediment loads, straight reaches, where produced by cutoffs, can rarely be made stable.

55. On the Rio Grande, which forms the international boundary line between the United States and Mexico for a distance of about 1,200 river miles, flood control has been accomplished in the vicinity of El Paso, Tex., by means of cutoffs and channel relocation over a river distance of 155 miles, which has been shortened to 88 miles. The new channel was so located as to make the aggregate area cut off at river bends from one country equal to that similarly cut off from the other country.

56. The system of artificial cutoffs being carried out on the lower Mississippi River differs from past experience in that the cutoffs are not isolated local improvements but have been made parts of a continuous river improvement. Slightly more than half the cost of the work done, to June 30, 1939, was expended in improving the reaches between cutoffs by dredging and auxiliary operations. Although the project as a whole has not yet reached a stage that permits this Committee to judge of the final outcome, it may be stated that at least four of the unfavorable conditions frequently encountered in making cutoffs elsewhere have not thus far manifested themselves: (1) The slopes in the cutoffs as well as in the reaches between cutoffs have not been steepened in excess of the slopes which naturally exist on the lower Mississippi River; (2) navigation has at no time been interrupted; (3) deposits of sediment in the channel below each cutoff, or below the entire series of cutoffs, have not

raised the river bed noticeably; and (4) in shortening the river's course, long, straight reaches have not been introduced. Appreciably increased flood-carrying capacity has resulted thus far, while at the same time the flood plain has been lowered. The methods used were made possible by conditions peculiar to the lower Mississippi River and are probably not applicable to rivers in general.

Other Methods of Flood Relief

57. Methods other than those previously discussed have been used to secure relief from flooding, and still other methods have been suggested or strongly urged by the general public.

58. Limitation of Use.—A large part of the damage from flooding, particularly in the cities, has resulted from making valuable improvements close to the river banks and directly in the flood plain where they naturally are subject to serious flood hazard. In the past, little control has been exercised over such construction. In the future, control of this practice may be exercised to a greater or less extent by means of municipal zoning laws, limiting the construction within the flood plain of buildings that would seriously retard flood flow. Furthermore, restriction of such lands to uses that will not obstruct flood flow usually will reduce flood losses. Studies in the flood protection of cities have shown that there are likely to be parts of the locality where flood relief can be secured most economically by utilizing the lands for parks and recreational purposes and allowing them to be submerged in great floods.

59. Flood-Plain Clearance.—In recent years there has been some sentiment for actually abandoning habitations in small communities located on the flood plains of rivers and moving the inhabitants to higher ground, thus clearing the flood plains and minimizing flood damage. The Reconstruction Finance Corporation recently offered to help finance the movement of individuals, industries, and even whole communities from frequently flooded bottom lands to perpetually safe highlands. In some cases on the Ohio River, citizens who formerly lived on the flood plain have been encouraged to resettle elsewhere in the town, and it is proposed to convert the lowlands thus evacuated into parks.

60. These instances point to a growing appreciation of the fact that it may frequently be more economical to remove habitations from flood plains than to provide flood control.

61. Flood Diversion.—Sometimes a disastrous flood peak can be averted by diverting part of the flood flow to some harmless or less harmful outlet. This type of relief is included in present plans for the improvement of the levee system on the lower Mississippi River. Here the diversion structures are of two types: Controlled weirs, such as the Bonnet Carre spillway, located above New Orleans, La., to limit the height of flood waters in contact with the levees protecting that city, and so-called fuse-plug levees, such as serve the New Madrid (Mo.) floodway, a short distance below Cairo, Ill. The floodways on the Sacramento River likewise have operated satisfactorily for a number of years. Floodways which traverse inhabited land involve a choice of evils and should be adopted only in cases where they are reasonably necessary to protect a preponderance of values.

62. On the lower Rio Grande, flood protection is accomplished for the valley lands by the diversion of the peak of disastrous floods from the river through artificial floodways of considerable capacity and length. The use of the lands occupied by the levees and floodways is covered by easements for the construction of levees and for the carrying of flood water through the floodways; and the owners of the lands are allowed to continue to use the land in the floodways for grazing purposes and any agricultural purposes which will not interfere with the use of the floodway for carrying flood water.

63. Forestry and Erosion Control.—There is a widespread belief that the cutting of forests has been an important element in the creation of great floods. Such evidence as is available, extending over two or three centuries in the United States and a much longer period abroad, indicates no marked tendency toward progressive change in climatic conditions, rainfall, or flood flows, although during the last century or two large areas have been denuded of forest cover. This being the case, it follows that no marked decrease could be effected in the crests of great floods by reforestation of the comparatively small amount of land that could be spared on the watershed of a great river. Forest cover, however, is beneficial in reducing minor floods and retarding erosion, particularly on steep slopes.

64. Flood damage is caused not only by the flood waters but also by the sediment and detritus carried, particularly where velocities are high. The accumulation of sediment brought down by floods may have an important bearing upon the life of reservoirs and flood-water channels. The prevention of erosion, therefore, is desirable, so far as it is economically feasible, from the standpoints of both reduction of flood damage and soil conservation.

65. Much may be accomplished by terracing, contour plowing, gully control, forest-fire prevention, and afforestation, but major floods cannot be averted thereby. Watershed conservation, using these words as a generic term to denote all that relates to preservation of the vegetal cover, to soil-erosion prevention, and to promotion of infiltration, however valuable it has proved in preventing the overloading of streams with sediment and detritus and in lessening the rapidity of concentration of storm runoff, does not constitute flood control, as it does not protect lands from overflow.

66. Prevention of Flood Damage.—A useful means of flood relief consists of avoiding, or reducing to a minimum, the damage consequent to flooding. This may be accomplished by evacuation of property, located on natural flood plains, which is easily damaged by water, or the occupancy of which imperils human or animal life unnecessarily. Permanent results along these lines may be accomplished through building and zoning regulations, through the removal of towns and industries to high ground as previously discussed under the heading "Flood-Plain Clearance," and through elimination of farm buildings from frequently flooded bottom lands. Prevention of flood damage thus effected is not a form of flood control. As an alternative, however, it is frequently preferable to flood control, because of economic considerations, because it eliminates unnecessary human suffering, and because, unlike flood control, it entails no subsequent operation and maintenance charges.

67. Prevention of flood damage calls for legislation to prevent villages and towns from being built on low ground and on islands subject to frequent overflow. Many villages and towns in the United States, thus situated, are not safe for human occupancy, and yet they cannot be protected against floods at reasonable cost. Their flood troubles, recurring frequently, cost federal and state agencies, in the form of emergency relief, sums entirely disproportionate to the value of the human interests and investments represented.

68. Forecasting and Flood Insurance.—An adequate flood-warning service is an important means of preventing flood damage to all classes of property in the path of floods and also may prevent the loss of many lives. Flood warnings, however, are not aimed at effecting the permanent removal of flood hazards, being principally of momentary value.

69. The forecasting of probable river heights with the approach of flood danger, and the prompt and frequent transmission of this information to residents and reservoir operators along the river, are important aids to flood protection, for it is the unexpected flood that is particularly costly to life and property.

70. Similarly, on colder watersheds, more information through snow surveys and frequent transmittal of the results thereof would inform operators of the potential hazards due to thawing.

71. In some cases the flood-relief problem might be solved most economically by providing insurance against floods, similar to fire insurance. The accumulation of sufficient data on probability to make such insurance practicable is worthy of consideration.

Scope of Engineering Studies and Reports

72. Besides determining the engineering structures which are adaptable to flood relief, and estimating the cost thereof, engineers should give thought to other related matters, such as the effect of flood reduction works upon riparian rights downstream in states where the doctrine of riparian rights is recognized. Probably most drainage areas in the United States on which such works may be desirable are developed to such an extent that any marked change in the regimen of the stream brought about by such works is likely to affect riparian rights adversely, where the latter are recognized. It would appear that this is an appropriate matter for engineering inquiry.

73. It is believed that both the profession and the public at large are beginning to realize that the waters of eastern drainage areas on which large populations may be located are becoming more valuable as development increases, and that any works built along such a stream, which may, to an appreciable extent, affect its regimen, are of prime importance to the local communities, as well as to the state and, therefore, should receive careful study.

74. In addition to the investigations already mentioned, and others which are commonly made, such as studies of flow records, flood frequency, flood-flow rates, and other characteristics of the floods of record, studies of the following matters are valuable: The development of the drainage area, both above and below the site of proposed flood-control works; the production and economic conditions on the watershed; and the net results to be derived from the works proposed.

Costs and Benefits

75. One of the most difficult problems to be solved in determining measures for flood relief is the limit of expenditures that would be justified. If costs exceed benefits, money is wasted. The justifiable expenditure should be based upon the best estimate of benefits that can be made.

76. Measure of Benefits.—In general, in order to justify an expenditure for flood relief, the benefits must be expressed in dollars. Intangible benefits may be added to the dollars and may serve to tip the scale, but an estimate of the tangible benefits in terms of dollars is necessary to success in most cases.

77. Prospective future losses from floods, if no flood-relief measures are taken, should be based primarily upon the events of the past, suitably modified to take future conditions into account. The difficulty lies primarily in the meager data relative to past flood losses. Under these circumstances reasonable expenditures by the federal government for the collection and publication of statistics of flood losses would seem to be amply warranted.

78. A serious aspect of many floods is the loss of life. There can be no doubt that protection to life is an important benefit from flood-relief measures and that it should be evaluated when costs and benefits are balanced.

79. Indirect Benefits.—In addition to preventing loss of life and protecting valuable property, both public and private, from damage, it is beneficial to protect against other losses resulting from paralysis of business, suspension of industry, interruption of transportation, unemployment, removal of the population, and arrested growth and prosperity of the locality, all sometimes chargeable to floods. Some of these losses are susceptible to fairly accurate measurement. In such cases the prevention of such losses becomes a tangible benefit.

80. There is scarcely a flood upon a small watershed seriously involving a single city that does not include some far-reaching losses resulting from interruption to railway service (often including the destruction of property in transit), affecting owners at distant places. It has been a noticeable feature of several great floods in the Ohio River Valley and elsewhere that great industries were entirely suspended, through-railroad transportation was cut off, and certain branches of trade were suspended, seriously affecting a large part of the United States. Similar losses of lesser degree occur every year at some place within the United States. Thus, the public in general becomes a beneficiary from flood-relief measures and, from the standpoint of equity, should participate in the costs thereof.

81. Magnitude of Expenditures.—The net cost of the relief works built by the Miami Conservancy District was close to $30,000,000. At that time (1920) the population of the nine cities and villages in the district which were the principal beneficiaries was 247,000. On this basis the per-capita expenditure was about $121. In this case the benefits from relief, determined from well-founded estimates, defended in court, amounted to about $3\frac{1}{3}$ times the cost of the relief works.

82. On the lower Mississippi River the total expenditures by the people of the valley for levees, to May 15, 1928, when flood control was taken over by the federal government, had been $292,000,000, or nominally $22.50 per acre,

on the basis of 13,000,000 acres (20,300 sq miles) of land susceptible to reclamation. While the benefits accrued largely to agricultural lands, they accrued also to cities and villages, and to railways, highways, and other lines of communication. The flood-protection system was not proof, however, against the great flood of 1927, which inundated 23,000 of the 30,000 sq miles comprising the alluvial valley and caused direct damage amounting to $263,334,400 (estimated by Mississippi Valley Flood Control Association). The flood-control plan adopted by Congress in 1928 provided for all-time protection of 20,550 sq miles[4] (13,152,000 acres) and partial, or part-time, protection of 9,450 sq miles[4] (6,000,000 acres) in addition. The total amount expended by the United States and local organizations to June 30, 1938, on flood control, channel improvements, and floodways was $640,643,440.

83. In 1937 the chief of engineers recommended the construction of a system of forty-five flood-control reservoirs on the tributaries of the Ohio River, in addition to those already authorized, at an estimated total cost, including lands and damages, of $246,000,000.[5] It was estimated that, if these reservoirs had been in operation during the 1937 flood, there would have been reductions in the flood stages amounting to 5.5 ft at Pittsburgh, 6.5 to 7.5 ft at Wheeling, W. Va., 3.3 to 4.3 ft at Cincinnati, Ohio, and 2.2 ft at Louisville, Ky. In order to protect centers of population from danger, even at the reduced crest heights, additional levees and flood walls would be needed by 155 communities at an estimated construction cost of $230,000,000, including land. The total for the additional reservoirs and the levees and flood walls is $476,000,000, which is about $200 per capita for all cities and towns on the Ohio River.

84. The per-capita expenditures mentioned in paragraphs 81, 82, and 83 are not comparable, except by taking into account the differences in character of the interests protected. In the Miami River Valley these are predominantly industrial, and in the Ohio River Valley, highly diversified. In addition, benefits of flood-control works in the Ohio River Valley are expected to extend, in some measure, to the lower Mississippi Valley, the population of which, however, has not been included in computing the foregoing per-capita cost.

85. In several cases, projects involving large expenditures for the local protection of municipalities have been effected through general bond issues, spread over the city concerned, or through a local taxing district. This method of financing is commonly employed for municipal storm-drainage projects, which, in the aggregate, compose the largest single item of expenditure for flood relief.

Agencies for Securing Flood Control

86. Until comparatively recent years, practically all flood-relief improvements were financed by the direct beneficiaries. Practically all farm levee districts and drainage districts have been built by direct assessments on the lands within the levees. The Mississippi River levees have been in the process of growth for more than two centuries. The federal government began to concern itself with the river in 1820. Its concern at that time resulted from

[4] Annual Rept., Chief of Engineers, U. S. Army, 1927; also, H. R. Doc. No. 90, 70th Cong., 1st Session.
[5] Com. on Flood Control, H. R. Com. Doc. No. 1, 75th Cong., 1st Session.

its interest in navigation, authorized by constitutional provisions relating to interstate commerce and the postal service.

87. It was not until March 1, 1917, however, that Congress enacted the first Flood Control Act, which authorized federal expenditures in the sole interest of flood control on the Mississippi and Sacramento rivers. Not until after the great Mississippi River floods of 1922 and 1927, which latter is considered to be the greatest on record, did Congress by Act of May 15, 1928,[6] place itself definitely on record as recognizing flood control in the alluvial valley of the Mississippi River to be a national problem. The need for defining the policy to be followed by the national government in prosecuting flood-control work was recognized in the Act of Congress approved June 22, 1936,[7] which embodies a "declaration of policy" as follows:

"Section 1. It is hereby recognized that destructive floods upon the rivers of the United States * * * constitute a menace to national welfare; that it is the sense of Congress that flood control on navigable waters or their tributaries is a proper activity of the Federal Government in cooperation with States, their political sub-divisions, and localities thereof; that investigations and improvements of rivers and other waterways including watersheds thereof, for flood-control purposes are in the interest of the general welfare; that the Federal Government should improve or participate in the improvement of navigable waters or their tributaries, including watersheds thereof, for flood-control purposes if the benefits to whomsoever they may accrue are in excess of the estimated costs, and if the lives and social security of people are otherwise adversely affected."

88. The National Flood Control Act of 1936 specifically authorizes some 270 flood-control projects, with an estimated construction cost of about $300,000,000, located in thirty-one states and affecting nearly every state in the Union. The projects authorized are to be prosecuted, under the direction of the Secretary of War and the supervision of the chief of engineers, U. S. Army. In addition to actual construction projects, the 1936 Act authorizes and directs the Secretary of War to cause preliminary examinations and surveys for flood control to be made in some 220 localities, and to continue surveys and studies of some eighteen reservoir sites.

89. By an Act of Congress approved June 28, 1938,[8] it was made incumbent upon the United States not only to pay the cost of lands, easements, and rights of way, but to maintain and operate all the works after completion. This amendment was made retroactive, so as to apply to all flood-control projects previously authorized by the Act of May 15, 1938, and by that of June 22, 1936, both as subsequently amended.

90. The Act of June 28, 1938, further provides that in any project where the construction cost of levees or flood walls can be substantially reduced by the evacuation of a portion or all of the area proposed to be protected, the chief of engineers may modify the plan accordingly, and at his discretion he may apply funds saved in construction costs toward the evacuation of the

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[6] Public No. 391, 70th Cong.
[7] Public No. 738, 74th Cong.
[8] Public No. 761, 75th Cong.

locality eliminated from protection and toward the rehabilitation of the persons so evacuated. This provision clearly indicates the trend of modern thought in flood control.

91. Practically all of the farm levee districts and drainage districts throughout the United States have been financed under state reclamation laws, through which all, or the greater part, of the cost has been assessed against the benefited property. As long as these levee districts were isolated, comparatively little harm resulted from the construction of the levees. The authority in general exercised by the states relating to this type of construction has been comparatively limited. In some places, upon the larger rivers, as already noted, extensive leveeing has resulted in the increased elevation of floods, due to the restriction of the flood waterway and the destruction of natural storage.

92. Experience with farm levee districts and reservoirs for flood relief has shown the necessity for coordination in this work. Within the past five years, extensive studies by the Corps of Engineers have made it plainly evident that no works for flood relief upon the large river systems of the United States should be undertaken without consideration by a governmental authority, competent to determine the effect of the improvement upon general plans for flood relief in the drainage area concerned.

93. With the general recognition that even local flood-relief projects tend to have far-reaching effects, either beneficial or detrimental, it becomes increasingly evident that a fair proportion of flood-relief costs should be borne by state and federal governments. It is believed to be important, however, that localities most directly benefited should bear an important part of such costs. Unless they do, local communities will demand unreasonable outlays, and expenditures for flood relief are quite likely to exceed the benefits to be obtained and become a serious burden on the taxpayer at large.

JOHN F. COLEMAN
HARRISON P. EDDY, JR.
L. L. HIDINGER
E. W. LANE
J. B. LIPPINCOTT
J. L. LYTEL
GERARD H. MATTHES
CHARLES B. BURDICK, *Chairman*

Committee of the Hydraulics Division on Flood Control

December 31, 1939

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

GRAPHICAL ARCH ANALYSIS APPLICABLE TO ARCH DAMS

Discussion

By Carl H. Heilbron, Jr., Assoc. M. Am. Soc. C. E., and William H. Saylor, Jun. Am. Soc. C. E.

Carl H. Heilbron, Jr.,[14] Assoc. M. Am. Soc. C. E., and William H. Saylor,[15] Jun. Am. Soc. C. E. (by letter).[15a]—The comments of Mr. Nelidov indicate that, in several instances, the writers have failed to offer sufficiently clear explanations. It is hoped that the questions raised will be answered satisfactorily herein.

The writers cannot agree with the suggestion that the convention whereby the centers of voussoirs and ends of voussoirs are used alternately "does not help to mechanize the process." In any method wherein all functions are determined at a single set of points it is necessary to average values at adjacent points in every step where integration is to be performed, and it is only by means of a convention such as that adopted that this averaging is avoided. Although the assumptions sound "complicated" when described, any one using them will realize that they result in the simplest possible set of operations.

In this connection Mr. Nelidov suggests the use of a larger number of voussoirs. It may be well to reiterate that in their example the writers, for simplicity, used a smaller number of voussoirs than they would recommend generally.

Mr. Nelidov identifies the "trial" forces with a similar set of forces which he calls "main" or "original." The latter, he says, are "known," meaning, it is presumed, that they can be obtained by processes not involving trial. Using his concept of "original" forces he arrives at a value of M_t' as (see Equation 20(*c*)) $M_t' = M_e + H_t\,y + V_t\,x$. He then mistakenly states that the writers define M_t (see Equation (21)) as $M_t = M_e + H_t\,y + V\,x$. Actually,

Note.—This paper by Carl H. Heilbron, Jr., Assoc. M. Am. Soc. C. E., and William H. Saylor, Jun. Am. Soc. C. E., was published in January, 1939, *Prcoeedings*. Discussion on this paper has appeared in *Proceedings* as follows: June, 1939, by I. M. Nelidov, Assoc. M. Am. Soc. C. E.

[14] Associate Engr., U. S. Engr. Office, Los Angeles, Calif.

[15] Asst. Engr., U. S. Engr. Office, Los Angeles, Calif.

[15a] Received by the Secretary December 19, 1939.

the writers define M_t as

$$M_t = M_e + H_t y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Thus, M_t is identical with M_t' except that V_t is taken as zero. That it is possible to let V_t be zero is due to the fact that the "trial" forces used by the writers are arbitrary values which are used as an aid in improving the accuracy of subsequent calculations, whereas the "original" forces considered by Mr. Nelidov are actual "known" forces which must be calculated. The advantage of the writers' concept of these forces is that they need not be calculated exactly but may be chosen arbitrarily as best suits the needs of the problem. In the case of V_t, it has been found practical to choose a value of zero so that this quantity is eliminated from the procedure.

Mr. Nelidov's question in regard to the effects of non-uniform temperature change is due, perhaps, to the writers' failure to explain that, in addition to taking the temperature gradient into account, of course, the average change of temperature must be used just as if the change were uniform.

The writers appreciate the fact that Mr. Nelidov brought these points to their attention.

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DISCUSSIONS

PROPOSED IMPROVEMENTS FOR LAND SURVEYS AND TITLE TRANSFERS

Discussion

BY PHILIP KISSAM, ASSOC. M. AM. SOC. C. E.

PHILIP KISSAM,[7] ASSOC. M. AM. SOC. C. E. (by letter).[7a]—There seems to be no question but that the methods involved in land transfer and land surveying records are in need of thorough revision. Co-operation between the engineering profession and the legal profession is essential if this objective is to be obtained. As Mr. Whitmore has stated, the Joint Committee may find that there is relatively little difference of opinion as to what should be done.

It may be remembered, however, that lawyers and engineers look at the problem from slightly different points of view. This results in a wide difference of opinion in regard to the purpose of land surveys and the requirements for land descriptions. The lawyer must have a survey and a description which identify the land. The engineer requires positive determination of the boundaries. At first glance these objectives may seem to be identical.

Suppose, for example, a residential lot were located in an area where the streets were not monumented but where there was a pair of arbitrarily placed survey control monuments for which the State plane co-ordinates had been determined accurately. To safeguard the title properly, the identity of the lot must be established. If street lines and distances from street intersections are mentioned (even if these lines are unmarked and quite indefinite), the lot is specifically identified. The streets mentioned and the rough location of the lots in the neighborhood are a matter of common knowledge; but, with such a description alone, the land surveyor cannot mark the boundaries. Thus such a description will satisfy the lawyer but not the engineer.

To locate the property properly, the best procedure would be to include, in the description, ties to the control monuments. These ties alone, with the dimensions of the lot, would locate it specifically. However, the title examiner would be unable to assure himself that the lot so described was, in fact, the lot

NOTE.—This paper by Philip Kissam was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1939, by George D. Whitmore, M. Am. Soc. C. E.

[7] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[7a] Received by the Secretary January 5, 1940.

for which he was searching the title, and such a description, although satisfying the engineer, would not be acceptable to the lawyer.

From these divergent points of view, in the opinion of the writer, spring the chief difficulties encountered in land transfer to-day and the abortive attempts to rectify them. Aside from these considerations, however, basic agreement between the two professions is not too difficult to obtain.

Mr. Whitmore emphasizes the fact that an important difficulty preventing the improvement in land-transfer procedure is the inertia that must be faced which tends to maintain the same systems hitherto utilized. Difficult as it is to institute change, a beginning has certainly been made. The work under Mr. Whitmore's direction in the Tennessee Valley is one of the beginnings of such a change. His work is based on sound principles which will pay dividends in the future. Again, Carl M. Berry,[8] Assoc. M. Am. Soc. C. E., has described another cadastral survey at the Grand Coulee Dam, based on these sound principles. Naturally, the first changes toward better methods will be found in the work of larger organizations and governmental units. State highway departments and county and municipal governments will utilize better methods for property surveys. On the other hand, it is expected that the general adoption of sound methods will not occur very rapidly but rather will grow outward from the centers established by the larger organizations, both public and private.

The great difficulty which prevents the more rapid adoption of good methods of property transfer, in addition to the inertia mentioned by Mr. Whitmore, is the lack of machinery for recording, properly, the results of land surveyors' work in the areas where conflicting survey evidence exists. Frequently a land surveyor must, of necessity, develop a solution for a certain area containing many parcels in order to establish the location of one particular parcel. There is no governmental agency which records the results of his work; nor is there simple machinery to obtain adjudication of his conclusions so that his work may not be lost nor later challenged by another surveyor. With public recognition of his work properly recorded, there would be a permanence of location and a reduction of survey repetition. It will be necessary, therefore, not only to overcome inertia, and divergent points of view between lawyers and engineers, but also to develop methods for recording land surveying data.

[8] "Combining Geodetic Survey Methods with Cadastral Surveys," by Carl M. Berry, *Proceedings*, Am. Soc. C. E., September, 1939, p. 1159.

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DISCUSSIONS

THEORY OF LIMIT DESIGN

Discussion

By J. A. Van den Broek, M. Am. Soc. C. E.

J. A. Van den Broek,[59] M. Am. Soc. C. E. (by letter).[59a]—It is gratifying indeed to note the large number of men who, by their generous discussions, have manifested their interest in the "Theory of Limit Design." As was to be expected, the views expressed reflected unfavorable as well as favorable opinions. The unfavorable opinions are the more valuable since they are likely to lead to the greatest clarification of the subject. Of these unfavorable opinions, some are the result of misunderstanding, and others of sincere disagreement. First, an attempt will be made to clear up possible misunderstandings, after which the disagreements will be discussed.

Mr. Sourochnikoff states: "Of course, if a few extreme fibers are broken at the supports, it does not mean that the entire beam will collapse." The theory of limit design does not envisage the breaking of any fibers anywhere at any time. In the realm of structural engineering, failure is the result of excessive deformations, and rupture is practically non-existent. Where non-ductile materials are used, the principles of limit design are non-operative.

Professor Fabian devotes his major attention to the opening sentence of the "Introduction"—namely, the statement that, "Traditionally all theory pertaining to the strength of materials and to structural design is based on two major assumptions: (1) That the material is elastic; and (2) that the principle of superposition applies." The theory of limit design breaks completely with these traditional assumptions. It would seem evident, from what has gone before, that as soon as the elastic limit at any point is exceeded the principle of superposition is violated. Professor Fabian, furthermore, claims that, "Where the conditions are such that the law of superposition cannot be applied, the distribution of loads to various members cannot be made on the

Note.—This paper by J. A. Van den Broek, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, and Francis E. Simpson; June, 1939, by Messrs. Joseph A. Wise, Alfred M. Freudenthal, Hans H. Bleich, Alfred S. Niles, and A. Floris; September, 1939, by Messrs. L. H. Nishkian, and F. G. Eric Peterson; October, 1939, by E. S. Fabian, Assoc. M. Am. Soc. C. E.; December, 1939, by A. A. Eremin, Assoc. M. Am. Soc. C. E.; and January, 1940, by L. H. Donnell, Esq.

[59] Prof., Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

[59a] Received by the Secretary January 8, 1940.

basis of simple statical conditions—namely, the force diagram—but it must be made on the basis of Maxwell's equations—that is, by the 'influence-line' method." No doubt it is agreed that Maxwell's equations are strictly within the limits of the theory of elasticity and are themselves predicated on the assumptions of elasticity and superposition. It is not clear, then, how they can be suggested as a tool in a field where the principle of superposition is non-operative and where the stresses exceed the elastic limit.

The principle of superposition, in the theory of elasticity, is generally assumed, except in case of buckling. It is practically never rigorously satisfied. The question as to whether or not the violation of the principle invalidates the theory built on it is one of degree. It has nothing whatever to do with the theory of equilibrium (statics), which is always true.

In the example of the steel tower (Fig. 15), the force diagrams were drawn on the assumption that the geometric proportions of the structure remained substantially unaltered. In this the practice usually followed is applied. It is justified on the basis of dictum (3), which states that deformations are of the order of magnitude of elastic deformations (that is, of secondary order of magnitude), under all loads up to the limit loads, but cease to be so when the limit load is exceeded.

Mr. Floris regrets the fact that only the fixed-end beam was analyzed; and, quite properly, Mr. Meursinge and Mr. Winter emphasize the fact that the capacity resisting moment of a beam is reached when the stress distribution is rectangular rather than triangular (Fig. 24(*c*) rather than Fig. 24(*a*)). Mr. Winter, furthermore, insists that deformation analysis on the basis of theory of limit design is essential, whereas Professors Wise and Niles deny that the strength criterion in structural design is deformation rather than stress.

The writer has the impression that most of the discussers had some familiarity with theories allied to the theory of limit design, if not with the latter theory itself. His paper, however, was directed primarily to the reader with little or no familiarity with the principles of limit design. For that reason, his main purpose was to present the major idea involved with the least possible mathematical or other complexities. A fixed-end beam is a theoretical abstraction rather than a reality. It was chosen to serve as an example simply because all its properties are thoroughly well known. For the same reason, the capacity moment was assumed to be the one obtained from the familiar formula, $M = \frac{S\,I}{c}$, rather than the more logical one obtained by Equation (2), which is strictly applicable only in cases of beams of symmetrical cross-section. In the case of a beam of irregular cross-section, the neutral axis shifts as soon as the elastic limit is exceeded, and Equation (2) should be written:

$$M = S_1 (\bar{y}_1 A_1 + \bar{y}_2 A_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

in which $A_1 = A_2$ (see Fig. 45). It was the desire to postpone considerations of detail such as these which lead to the assumption that the capacity-bending moment might be expressed by Equation (2). The writer stands corrected. He realizes that this is a consideration of insufficient merit and recommends

that capacity moments be computed by Equation (2), or, in the case of beams with unsymmetrical cross-section, by Equation ($2a$).

As to deformations of beams subject to ductile yielding, Mr. Winter supplies some theory and certain very interesting test results. A very close agreement between theoretical and test results involving the ductile behavior of beams may be observed in Fig. 46.[60] If the deflection under a limit load (when the stress distribution resembles that shown in Fig. 24(c)) is compared with the deflection obtained when the beam behaves completely elastically under the same load, the ratios of these deformations are found to be: For the rectangular beam (Fig. 46(a)), 24/16 = 1.50; for the circular beam (Fig. 46(b)), 23/17 = 1.35; and for the wide-flange beam (Fig. 46(c)), 38/31 = 1.25. This evidence serves as proof of the claim that the deformations are of the order of magnitude of elastic deformations until the last redundant (the fibers at the neutral axis) members have reached their elastic limit.

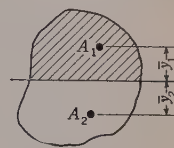

FIG. 45

Professor Wise is the only reviewer who questions the validity of the three dicta under the heading "Theory." It is conceded that without these dicta the entire theory of limit design collapses. As to the first dictum, Professor Wise calls it vague. The writer merely wanted to emphasize what he regarded as a well-known special aspect of engineering structures. The third dictum Professor Wise dismisses with a single word "incorrect." On the contrary, the writer regrets that he could not emphasize it more strongly by means of italics or capital letters, as he regards it as the very cornerstone of the theory of limit design. To test dictum (3) by Professor Wise's definition of statically indeterminate structures: He states, "The essential characteristic of a statically indeterminate structure is that it has redundant members, reactions, or restraints in excess of the number just sufficient to maintain static equilibrium." Suppose, then, that such superfluous redundant members are all stressed beyond the elastic limit. By Professor Wise's definition, sufficient members are left to form an elastic system in equilibrium. This system, as stated in the "Synopsis," behaves elastically. Hence the statement in dictum (3) that one member in the primary system (that is, one member in addition to the redundant members) must exceed the elastic limit, or buckling strength, before deformation in excess of elastic deformation will take place. It would involve extra diagrams to prove the point mathematically, but the writer suggests that the reader take any redundant structure, beam or truss, or a combination of both, and proceed to find the displacement at some point. He will then probably apply an auxiliary load at this point in the direction of the desired displacement and compute the resulting auxiliary forces in the structure. When he does so, however, he will, no doubt, in perfect conformity with the rules of the theory of elasticity, remove as many reactions, restraints, or members (in other words, redundants) as is consistent with stability. In his computations for the desired displacement, the redundants do not appear, since the forces therein, induced by the auxiliary load, are all zero. The

[60] "Deformations of Beams Involving Ductile Behavior," by E. O. Scott; thesis presented to the University of Michigan in 1939 in partial fulfillment of the requirements for the degree of Master of Science in Engineering.

displacement of a point in a redundant structure thus appears as the displacement of a point in a statically determinate elastic system. Hence, so long as the primary system is intact, deformations are of the order of magnitude of elastic deformations, which is all that dictum (3) states.

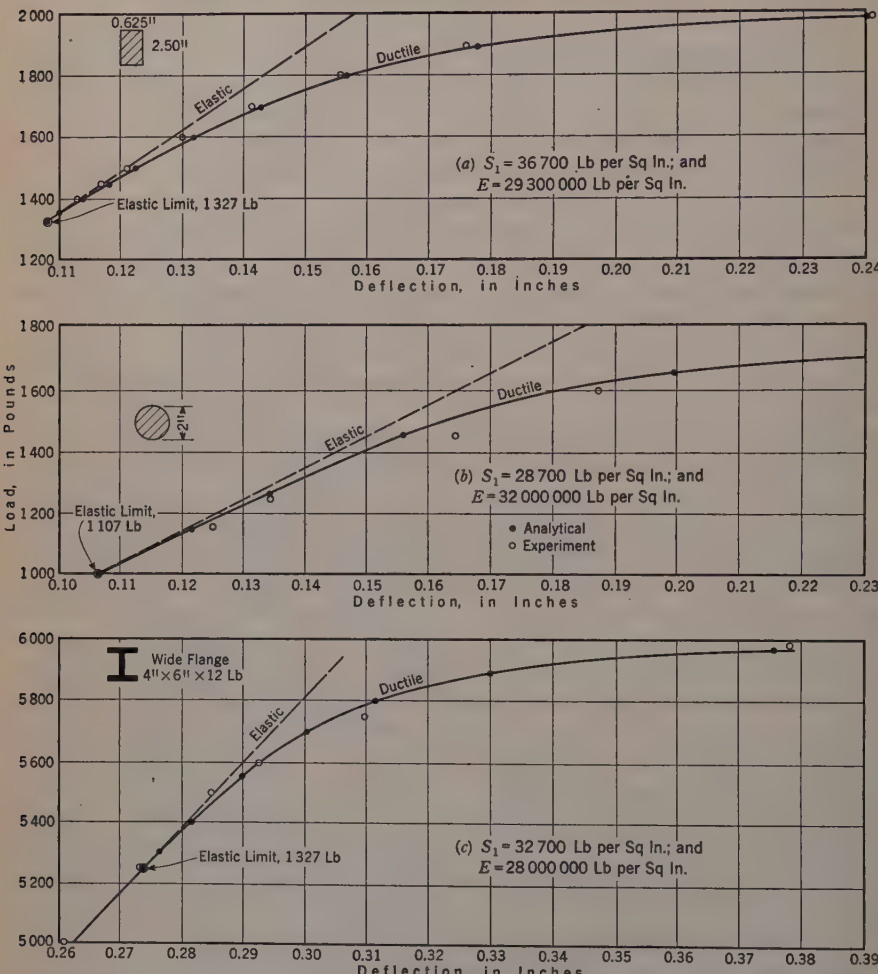

FIG. 46.—COMPARISON BETWEEN THEORETICAL AND TEST RESULTS INVOLVING THE DUCTILE BEHAVIOR OF BEAMS

There are shelves full of treatises on the theory of elasticity. The striking revelation to the practicing engineer is that the elastic deformations of a structure, in themselves, are only rarely of interest, and then generally only of secondary interest. In structural engineering the theory of elasticity is a supplementary theory. It serves the purpose (area moments, slope deflection, conjugate beams, least work, elastic energy, etc.) of writing independent equations to supplement those obtained from statics. Ask any engineer the

probable stress in a modern railroad bridge, and he will give an immediate and correct answer. Ask him the probable deflection of the same bridge under load, and he is likely to want paper, pencil, dimensions, etc. The point is that, for structures of normal proportions, deflections are safe, so long as they are of the order of magnitude of elastic deflections. Failures, generally, are not the result of rupture but of excessive deformations—deformations of an order of magnitude greater than elastic deformations. In the laboratory specimens are tested and actually broken. A structural steel beam is never broken either in the laboratory or in the field. Under load it may be distorted out of shape to the point where it becomes a mass of wreckage, but in spite of the designer's emphasis on strength and on stresses, in the last instance, it is only deformation that really counts.

The writer did not state, as Professors Wise and Niles infer, that the theory of limit design is predicated upon permissible deformations, as distinct from the theory of elasticity, which is supposed to be predicated upon permissible stresses. In the "Introduction," prior to the introduction of Fig. 2, it is stated that "* * * it is proposed to continue to rely on the criterion that always was the primary one, the criterion of permissible deformations." In a statically determinate beam in a building, the designer generally is not concerned about deformations so long as the stresses remain within the elastic limit. Dictum (3) maintains that, in a redundant beam, even when the redundants have become completely ductile, the deformations are not of a magnitude that would give the structural engineer especial concern, because the primary

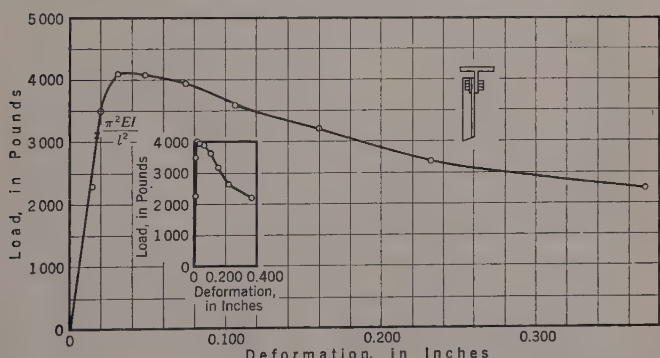

FIG. 47

system still remaining behaves in the manner of a statically determinate, purely elastic, beam with deformations of relatively small order of magnitude.

Mr. Meursinge calls attention to the importance of connections. Specifications emphasize the importance of the property of ductility, but in order that this property may be made effective, the connections must be so designed that the yield point in the main part of the member is exceeded before the connection fails. In the discussion of the tower (Fig. 15) the connections were assumed to be welded in order to keep the argument simple and postpone the consideration of complexities.

Elsewhere[61] the writer has shown the results of a series of tests on $1\frac{3}{4}$-in. by $1\frac{3}{4}$-in. by $\frac{1}{8}$-in. angle irons such as are commonly used as diagonal bracing in transmission towers (see Fig. 15). Bolt connection of an angle iron through one leg weakens the iron on two counts—the reduction of net area and the

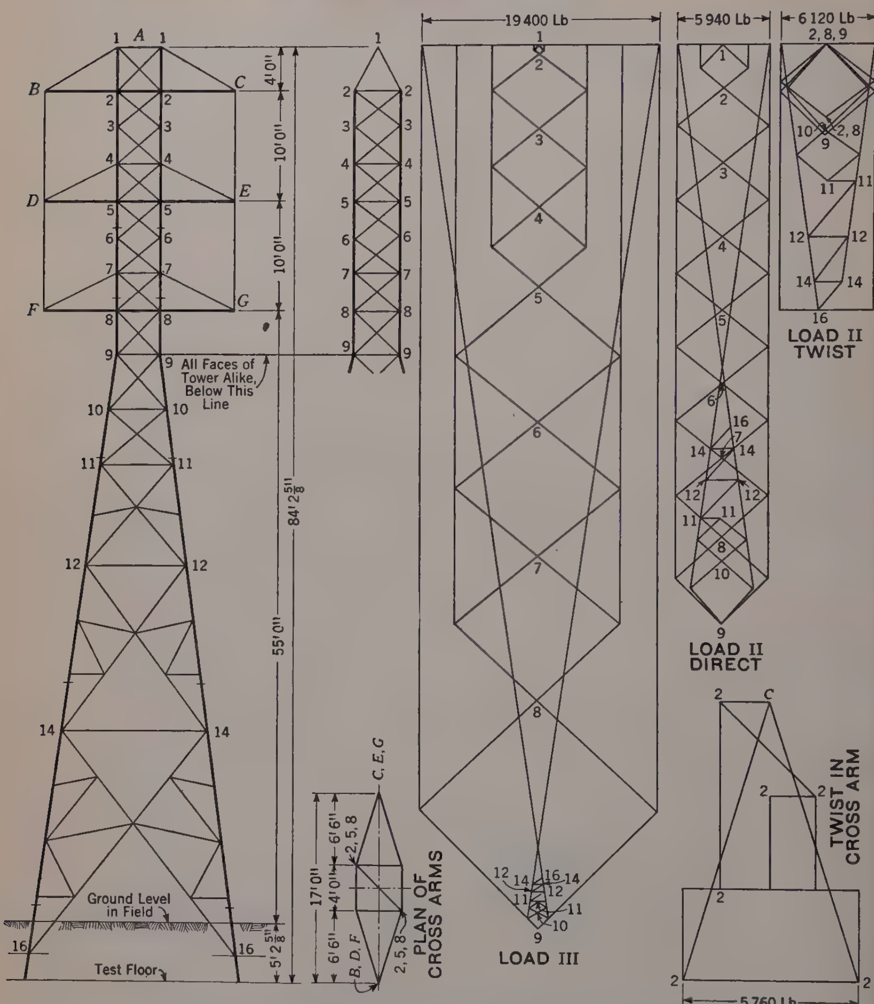

FIG. 48.—STRESS ANALYSIS OF A FULL-SIZE, 55-FOOT TRANSMISSION TOWER

eccentricity of the applied load. By perforating both legs of the iron with extra holes, the strength of the iron is somewhat increased and the ductility of the member is augmented in proportion to the number of extra holes. Different types of end connections of angle irons and different degrees of perforation show marked differences in strength and even greater variations

[61] "Effect of Connections and Rivet Holes on Ductility and Strength of Steel Angles," by J. A. Van den Broek, *Civil Engineering*, February, 1940, Figs. 1 to 6, p. 83 (see also Table 1).

TABLE 4.—STRESS TABLE CORRESPONDING TO FIGURE 48
(Stresses in Pounds per Square Inch)

Member	Description	Dimensions, one angle	Maximum working stress, S	ULTIMATE TEST STRESS, U, FOR THE FOLLOWING ULTIMATE LOADS (SEE TEXT)						Ratio, $\frac{U}{S}$
				I	II Direct	II Twist	III	IV	Maximum	
1–1	Guy-wire support	6 by 6 by $\frac{3}{8}$	± 300	....	....	....	± 700	....	± 700	2.34
1–2	Peak angle	$2\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{3}{16}$	± 3 600	+2 500	± 3 600	....	± 400	+ 100	+ 6 600	1.83
1–2	Peak angle (diagonal)	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 300	....	....	....	± 600	....	± 600	2.00
1(A and B)	Vertical hanger	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	− 3 500	−6 300	....	....	....	....	− 6 300	1.80
B(6–2), DE5, and FG8	Cross arm	$3\frac{1}{2}$ by 3 by $\frac{1}{4}$	+ 7 600	+3 300	± 9 900	....	± 3 800	....	+ 17 000	2.14
2–2, 5–5, and 8–8	Normal face: Strut	$3\frac{1}{2}$ by 3 by $\frac{1}{4}$	+ 5 500	+3 300	....	+6 600	....	....	+ 9 900	1.80
2–2, 5–5, and 8–8	Parallel face: Strut	3 by $2\frac{1}{2}$ by $\frac{1}{4}$	+ 1 900	+1 100	....	+1 600	± 1 100	....	+ 3 800	2.00
2–2, 5–5, and 8–8	Horizontal, diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 2 600	....	....	±4 700	....	....	± 4 700	1.80
2–5	Leg angle	3 by 3 by $\frac{1}{4}$	+14 900	+3 800	+15 400	....	+17 000	+ 400	+ 36 600	2.46
2–3, 3–4, and 4–5	Normal face: Diagonal	$1\frac{3}{4}$ by $1\frac{1}{4}$ by $\frac{3}{16}$	± 3 400	....	....	±3 900	± 5 100	....	± 9 000	2.65
2–3, 3–4, and 4–5	Parallel face: Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 4 800	....	± 4 000	±3 900	....	....	± 7 900	1.65
4–4, 7–7	Normal face: Strut	$1\frac{3}{4}$ by $1\frac{1}{4}$ by $\frac{3}{16}$	− 1 800	−3 300	....	....	....	....	− 3 300	1.84
4–4, 7–7	Parallel face: Strut	$1\frac{3}{4}$ by $1\frac{1}{4}$ by $\frac{3}{16}$	+ 600	+1 100	....	....	....	....	+ 1 100	1.84
4D, 4E, 7F, 7G	Vertical hanger	$1\frac{3}{4}$ by $1\frac{1}{4}$ by $\frac{3}{16}$	− 2 000	−3 600	....	....	....	....	− 3 600	1.80
5–8	Leg angle	4 by 4 by $\frac{9}{32}$	+32 200	+5 400	+30 400	....	+48 600	+ 700	+ 85 100	2.64
5–6, 6–7, and 7–8	Normal face: Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 4 900	....	....	±3 900	± 8 800	....	± 13 200	2.69
5–6, 6–7, and 7–8	Parallel face: Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 5 100	....	± 4 000	±3 900	....	....	± 7 900	1.55
8–11	Leg angle	5 by 5 by $\frac{5}{16}$	+41 300	+5 400	+33 700	....	+73 200	+1 000	+113 300	2.75
8– 9	Normal face: Diagonal	2 by 2 by $\frac{3}{16}$	± 6 100	....	....	±4 300	±13 560	....	± 17 800	2.92
8– 9	Parallel face: Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 5 600	....	± 4 300	±4 300	....	....	± 8 600	1.54
9–10	Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 4 000	....	± 2 700	±3 900	± 1 000	....	± 6 600	1.65
10–11	Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	± 2 900	....	± 2 000	±2 800	± 700	....	± 4 800	1.65
11–11	Strut	2 by 2 by $\frac{3}{16}$	+ 2 100	....	+ 1 500	+1 800	+ 500	....	+ 3 300	1.57
11–14	Leg angle	5 by 5 by $\frac{5}{16}$	+43 000	+5 400	+32 200	+3 800	+72 000	+1 500	+114 900	2.67
11–12	Diagonal	$1\frac{3}{4}$ by $1\frac{3}{4}$ by $\frac{3}{16}$	− 5 400	....	− 3 500	−5 000	− 1 200	....	− 8 500	1.57
12–12	Strut	$2\frac{3}{4}$ by $2\frac{3}{4}$ by $\frac{3}{16}$	+ 3 400	....	+ 1 900	+2 700	+ 800	....	+ 4 600	1.35
12–14	Diagonal	2 by 2 by $\frac{3}{16}$	− 4 800	....	− 2 600	−3 900	− 900	....	− 6 500	1.35
14–14	Strut	3 by 2 by $\frac{3}{16}$	+ 3 200	....	+ 1 500	+1 800	+ 500	....	+ 3 300	1.03
14–16	Leg angle	5 by 5 by $\frac{5}{16}$	+40 600	+5 400	+27 600	+1 900	+71 300	+2 400	+108 600	2.67
14–16	Diagonal	$2\frac{1}{2}$ by 2 by $\frac{3}{16}$	− 4 300	....	− 2 000	−2 600	− 700	....	− 4 600	1.07

FIG. 49.—TOWER SUBJECTED TO A LOAD OF 180% OF DESIGN VALUE

in ductility. Fig. 47 shows the load-deformation relationship of a $1\frac{3}{4}$-in. by $1\frac{3}{4}$-in. by $\frac{1}{8}$-in. angle with a slenderness ratio of 200 in compression. Figs. 48, 49, and 50 illustrate the results of a test of a full-size transmission tower, 55 ft high.

FIG. 50.—ENLARGED VIEW OF THE FAILURE (SEE FIG. 49)

Stress diagrams are shown in Fig. 48, with corresponding stresses in Table , the ultimate loads being identified as follows:

I. Vertically downward
   (*a*) 2 160 lb at Point *A* = 180% of design load
   (*b*) 3 240 lb at each of points *B*, *C*, *D*, *E*, *F*, and *G* = 180% design

II. Horizontal parallel
   (*a*) 6 120 lb at Point *A* = 180% of design load
   (*b*) 5 760 lb at Point *C* = 180% of design load

III. Horizontal normal

(*a*) 1 300 lb at Point A ≐ 306% of design load
(*b*) 7 290 lb at Point B = 424% of design load
(*c*) 7 290 lb at Point C = 610% of design load
(*d*) 5 730 lb at each of points D, E, F, and G = 333% of design load

IV. Weight of tower = 9 600 lb

Fig. 49 shows the tower loaded with an 80% over-load, and Fig. 50 shows a close-up of the failure in the diagonal bracing of the top panels. Fig. 51 shows a close-up of the failure of one of the 4-in. by 4-in. by $\frac{3}{8}$-in. legs of a heavy suspension tower under the following loading:

Vertical load = design load + 80%
Longitudinal load = design load + 80%
Transverse load = design load + 180%

The panels of this tower were each 10 ft high, and at failure the structure was sustaining the loads shown in Table 5.

TABLE 5.—Loads on Test Tower, at Failure

Loads	Guy wire	Top arm	Mid-arm	Bottom arm
Vertical	1 620	3 060	4 500	4 500
Longitudinal	4 680	5 400	....	....
Transverse*	4 600	5 460	7 160	7 160

* The wind load was compensated by an addition of 750 lb to the transverse load at each of the four points.

The most serious disagreement revealed by the discussion is the opinion advanced by Messrs. Bleich, Freudenthal, Mirabelli, and Wise to the effect that the philosophy of limit design is not applicable in cases where buckling of members may be encountered.

For some twenty years the writer has talked about a theory of ductility ("plasticity" is a term applicable to structural steel only when it is red hot). In every course he gives on the theory of elastic energy the lectures are closed with a discussion of the limitations of the theory of elasticity. On one such occasion, in 1924, at the termination of a course in Detroit, Mich., Mr. Goodrich was present. It was then that the writer learned that he, with his theory of ductility, and Mr. Goodrich with his design practices, talked essentially the same language. After innumerable discussions with Mr. Goodrich, the term theory of ductility was abandoned and the term "theory of limit design" was substituted, so as to make the title applicable to trusses involving members subject to buckling. Whereas some men (Professors Mirabelli and Wise) merely question the application of the theory of limit design in cases where buckling may occur, others (Messrs. Bleich and Freudenthal) conclude definitely that such application should not be permitted.

The writer's reply is two-fold. The first is that Fig. 47 shows that although the load-deformation relationship for a member in compression is not the

FIG. 51.—FAILURE OF ONE OF THE LEGS OF A HEAVY SUSPENSION TOWER

idealized curve shown in Fig. 2, it appears nevertheless that, when the deformation is eight times as great as that shown when the Euler's buckling strength is first reached, the strength of the column is still of the order of magnitude of Euler's value. This eight-fold deformation under substantially a constant compressive value is generally more than enough to insure the functioning of limit-design principles.

The second answer is: Notwithstanding the warning of Messrs. Bleich and Freudenthal that the theory of limit design should not be applied to trusses, it has been applied very successfully for thirty years to the design of transmission towers of which Fig. 51 is an example. Mr. Goodrich states that his first design was condemned by five eminent authorities. It may be added that his present designs would still be condemned by other eminent authorities if they were judged by conventional design principles based on the theory of elasticity (in the paper, see "Design of a Steel Tower: Comment on Steel Tower Design," last two paragraphs).

Mr. Silverman laments the fact that the most complicated structures are analyzed on the basis of facts obtained from simple test specimens. He regrets that it is generally not feasible to use the entire structure for tests. The writer concurs in these views and on the strength of them concludes that the design of transmission towers is at present in the most advanced and most wholesome state. For thirty years transmission towers, designed by the theory of limit design, have been sold by at least one plant on the basis of actual load-resisting performance rather than on the basis of satisfying certain criteria relative to mere elastic behavior. These tests have demonstrated that the theory of limit design makes it possible to predict the strength of towers to within $\pm$ 10 per cent. Other plants have followed the same example with the result that probably most transmission towers in the United States and Canada are of insufficient strength as tested by theory-of-elasticity standards; but, actually, they exhibit sufficient strength to carry the design load plus a stated over-load factor.

Fig. 48 and Table 4 are significant in that they show the complete stress analysis of a complicated three-dimensional redundant structure under three distinct types of loading. The simplicity of the analysis, substantially the same as that of Fig. 15 in the paper, is not the result of compromise and does not involve a sacrifice of accuracy as compared with standards established by stress analysis based on Hooke's law. On the contrary, it has a two-fold merit: First, that of great simplicity; and second, the merit of very satisfactory agreement with observed facts, as established by innumerable tests. It is conceded that the load-deformation relationship of columns beyond the buckling load is an involved one, affected as it is by end-connection, slenderness ratio, and eccentricity of applied load. In spite of innumerable tests on columns, which have been recorded, this particular relationship appears practically never to have been measured. Before limit-design principles will be applied with confidence to any type of truss, other than transmission towers, such data should be obtained for every type of compression member under every type of end connection and manner of loading.

Mr. Meursinge, very aptly, quotes Professor Kist to the effect that Hooke's law and theory of ductility are both "simplifications of a very intricate reality * * *. Both may be compared to cartoons, because they both 'express reality in a very primitive manner.'" The writer's only hope is that the theory of limit design is not too much of a caricature. Quite properly, Professor Niles warns against using limit-design principles where materials are involved which do not behave in agreement with Fig. 2. Wood, concrete, and earth do not obey Hooke's law rigorously. Nevertheless, the designer commonly applies strength-of-materials formulas to them, which are based on Hooke's law. This procedure is fully justified, of course, when a proper factor of safety is used. Similarly, limit-design principles may be applied, with reservations, to structures built of materials that are imperfectly ductile. The reservations naturally become prohibitive when the material is completely lacking in ductile properties.

It was not due to the theory of elasticity, but rather to the good sense of structural engineers (possibly without the aid of any theory to which a label could be attached, but nevertheless the result of sound logic and correct observation) that such materials, completely devoid of ductile properties, were never used for structures such as bridges, towers, buildings, or airplanes.

Mr. Silverman refers to Timoshenko,[21] even as does Mr. Eremin. Mr. Silverman, however, correctly attributes the original contribution not to Professor Timoshenko but to Mr. Hencky. Furthermore, Mr. Silverman points out that the agreement of Mr. Hencky's contribution with the theory of limit design is incidental, which is very gratifying. Mr. Hencky's contribution is offered as an attempt to explain what Mr. Silverman calls the Bauschinger effect—namely, that the effect of cold working on steel in tension is to raise the elastic limit in tension and to lower it in compression. This so-called Bauschinger effect has two serious flaws. In the first place, Bauschinger never made this claim; and in the second place, the claim when made is not true. (See "The Effect of Cold Working on Elastic Properties of Steel,"[62] in which an entire chapter—entitled "Cold Stretching Steel Does not Lower Its Elastic Limit in Compression"—is devoted to refuting this commonly found erroneous statement.)

Professor Donnell's discussion is very much appreciated as being most constructive. He "hits the nail on the head" when he states: "There have been many theoretical and experimental studies of buckling problems in both the elastic and plastic range; but, heretofore, this particular phase of the question [the relation between longitudinal resistance of struts and the longitudinal deformation during buckling] has not been considered important and has apparently escaped investigation."

The theory of elasticity has not only dominated methods of design but has controlled research as well. To return to the original thesis: If the theory of elasticity is so excellent, why not specify purely elastic material; and, *vice*

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[21] "Zum Theorie Plastischer Deformationen und der hierdurch im Material Hervorgerufenen Nachspannungen," by H. Hencky, *Zeitschrift für angewandte Mathematik und Mechanik*, 1924, p. 323; also, "Strength of Materials," by S. Timoshenko, Part II, 1930, p. 668.

[62] "The Effect of Cold Working on the Elastic Properties of Steel," Carnegie Scholarship *Memoirs*, Iron and Steel Inst., Vol. IX, 1918, p. 138.

versa, if the engineer realizes that he cannot possibly build with purely elastic material, then why not include the property of ductility in design theory?

The writer is unable to accept two alleged statements of fact made by Mr. Freudenthal. In Section (2) of his discussion Mr. Freudenthal states that "Instantaneously applied loads cause the upper yield point to drop." In the writer's opinion the reverse is the fact. In structural steel, the corner at Point *B* as shown in Fig. 2 is the sharpest when loads are applied slowly. In Section (3) Mr. Freudenthal states: "The occurrence of pulsating and reversed stresses will always demand the design of bridges on the basis of the endurance limit of the material." Reversal of stresses in bridges is so rare as to constitute the exception rather than the rule. Furthermore, although capacity loads for certain details[63] (for example, stringer connections to floor beams) may occur each time a heavy locomotive passes over the bridge, capacity loads for the entire truss may not occur oftener than once or twice in the life of the bridge. Therefore, although fatigue must be considered in connection with bridge design, it is of secondary and not of primary importance.

The writer desires to express his indebtedness to The Canadian Bridge Company, Ltd., for permission to reproduce illustrations, to Mr. Goodrich for his unfailing interest, and to the Horace H. Rackham School of Graduate Studies of the University of Michigan for aid received in providing assistance and in furthering tests herein recorded.

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[63] "Fatigue Tests on Connection Angles," by W. M. Wilson, M. Am. Soc. C. E., and John V. Coombe, Jun. Am. Soc. C. E., *Bulletin*, Univ. of Illinois, Vol. XXXVII, No. 6, Series No. 317.

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DISCUSSIONS

LARGE CORE DRILLS AID CONSTRUCTION AT CHICKAMAUGA DAM

Discussion

By James S. Lewis, Jr., Assoc. M. Am. Soc. C. E.

James S. Lewis, Jr.,[7] Assoc. M. Am. Soc. C. E. (by letter).[7a]—The discussions have revealed an appreciation of the difficulties often encountered in obtaining satisfactory foundations in limestone. Extensive weathering, solution, erosion, and geologic disturbance frequently have combined to produce a condition which is not disclosed by generally accepted methods of exploration. The erratic results obtained from small borings may prove very misleading and difficult to interpret, and even large borings will leave much essential information to be revealed by the excavation.

As emphasized by Mr. Hays, the services of a geologist skilled in interpreting the information obtained from borings were used to excellent advantage. The aid rendered by Mr. Fox proved invaluable. Mr. Floyd indicates the value of large core holes as shafts for various purposes, and their usefulness also has been demonstrated in mining operations where it was necessary to attain great depths.

There is no gainsaying Mr. Newsom's statement to the effect that large drills are capable of higher development. This is true of shot drills of all sizes, as they have not enjoyed the mechanical refinements that have been applied to the best diamond drills. A reduction in weight would afford greater ease of handling over the rough terrain that must frequently be traversed, and a closed type of construction would prevent the damage that follows when shot get into the moving parts. An automatic shot feed, powered by a standard water meter, was developed at Norris Dam. The immediate effect was a reduction in shot consumption to 25% of the former quantity, a smooth core and hole of uniform size, and less wear on the drilling tools. There is still opportunity for great improvement.

Note.—This paper by James S. Lewis, Jr., Assoc. M. Am. Soc. C. E., was published in June, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1939, by J. B. Newsom, Esq.; and January, 1940, by Messrs. J. G. Tripp, James B. Hays, and O. N. Floyd.

[7] Asst. Constr. Supt., Watts Bar Dam, TVA, Spring City, Tenn.

[7a] Received by the Secretary January 16, 1940.

Different methods of obtaining a foundation might have been chosen, as suggested by Mr. Tripp. The writer doubts, however, that the caisson used in the rotation water method could have been used economically to cut the limestone encountered for the distances required.

In describing the use of large core drills for sinking shafts under extremely adverse conditions, the writer's chief desire was to reveal the adaptability of the equipment when the drilling was supplemented by grouting. It is obvious that the methods described would have been impractical without sealing the rock and overburden against excessive leakage.

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DISCUSSIONS

BEACH EROSION STUDIES

Discussion

By Charles T. Leeds, M. Am. Soc. C. E.

Charles T. Leeds,[10] M. Am. Soc. C. E. (by letter).[10a]—The author has given an excellent syllabus of beach erosion study with an enlightening commentary thereon. Beach protection is a most fascinating and a most tantalizing field of engineering, because of its many elusive and often indeterminable factors. In a field of science which is so largely empirical the wise engineer will search all possible sources of information. Therefore, such a general outline as the author gives of a typical study, together with an explanation of the need of each item, is a valuable contribution. Because this field of engineering is so largely based on experience, however, one of the greatest needs is a more extended and detailed record of results under varying conditions, but perhaps such record should be made the subject of another paper.

The engineer who attempts to diagnose the symptoms of an inadequate or menaced beach, and to prescribe remedial measures, is in much the same position as a doctor prescribing for a human patient; for a beach, like a human being, is ever-changing—continually wasting and rebuilding. The first requisite is the past physical record (geologic as well as historic) and all available past surveys and photos should be obtained and supplemented by personal observations, past and present. Even then the records are often too meager and intermittent to afford other than a very broken story of the changes that have taken place in the shore line and in the offshore conditions.

Just here a word of caution is important. Not only the year of the survey, but also the season, should be learned, if possible, for the direction of littoral drift may vary with the time of year. Unfortunately there probably will be no record of the weather at the time of these old surveys; yet this is important, for the beach may recede a considerable distance in a single storm, and in a succeeding period of fair weather it may undergo a more or less complete

Note.—This paper by Earl I. Brown, M. Am. Soc. C. E., was presented at the meeting of the Waterways Division at Jacksonville, Fla., on April 21, 1938, and published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Morris N. Lipp, M. Am. Soc. C. E.; May, 1939, by Messrs. George A. Soper, and James J. O'Rourke; and June, 1939, by Messrs. Elliott J. Dent, and Ralph F. Rhodes.

[10] Maj., Corps of Engrs., U. S. Army (*Retired*); Cons. Engr. (Quinton, Code & Hill-Leeds & Barnard), Los Angeles, Calif.; State Consulting Seacoast Engr.

[10a] Received by the Secretary December 14, 1939.

restoration by natural accretion. Therefore, a given survey or photograph may or may not be representative of the average shore conditions. This is especially worthy of remembrance in case of a dispute regarding title to lands bordering the ocean, for in some instances the land values in question may reach huge proportions.

Where there has been a considerable lapse of time between surveys it is highly important to use these with great care. It is by no means certain that the movement of the shore line between the locations shown on different dates has been either continuous or all in the same direction.

The writer recalls an engineer friend who compared two surveys dated, say, 1860 and 1890, and because the river mouth in 1890 was 1 000 ft westward of its location in 1860, he assumed a progressive westward littoral migration of 30 to 35 ft per yr, notwithstanding that the present prevailing littoral drift is in the opposite direction. He assumed, from recorded statements of "old-timers," that southeasterly storms were more frequent and more severe in the Nineteenth Century than they have been in the Twentieth Century, and therefore, that the predominant littoral drift has been reversed. Such a theory of meteorological cycles is not believed to be tenable. It is more probable that the river mouth actually migrated eastward under the influence of prevailing waves from the southwest, until a heavy flood broke out a new mouth well toward the west, when it again resumed its eastward travel. Such a migratory cycle is well known to those familiar with inlets on sandy coasts where there is a predominant littoral drift. The survey of 1860 probably showed the river mouth at nearly the eastward limit of its migration during a dry period, whereas the survey of 1890 probably showed its position just after a heavy flood, at the beginning of its migratory cycle.

In discussing the need for study of past wind and weather conditions, the author might well have given even greater emphasis to the statement that "If it is apparent that there is a great seasonal variation in wind direction and intensity, and if the detail of the study warrants, such wind-roses should be plotted for each month." The writer has seen (as have doubtless many others) a single storm, in a short time, undo the work of the prevailing wind movement over a long period. Such information will not receive adequate attention in a wind-rose showing the aggregate of all observations over an extended period.

A very important item of information, of which there is seldom adequate accurate record, is that of wave data. It is true that the author states (see heading "Field Work: Wave Data"): "Whenever observations on currents in the ocean or sand in suspension are made, careful readings of wave data should be made and recorded." However, such records can be only fragmentary and are pertinent only for the times of the "observations on currents in the ocean or sand in suspension." Wave action undoubtedly constitutes the major factor in littoral sand movement; therefore, the wave direction, velocity, height, and especially duration are even more important than the wind direction and velocity. Indeed, serious beach erosion is sometimes caused by severe wave action occurring in periods when there is no appreciable local wind, these waves being the result of distant storms.

As the author states (see heading "Scope of a Beach Erosion Study: Littoral Drift"): "* * * the direction of predominant littoral drift * * * is most important in the design of a protective system," and "If the detail of the study permits, observations as to the direction of littoral drift should be continued over a period of a year to determine these seasonal changes." Yet current measurements and sand samplings are admitted to be impracticable, except under relatively calm conditions, and to continue them over an extended period, such as a year, is often unduly expensive.

What is needed is the summation of all movement of the sand, landward or seaward, and upcoast or downcoast. By placing, on the beach, some material of approximately the same specific gravity as the beach sand or gravel, but easily distinguishable therefrom, and by observing the location of this material periodically, the aggregate movement can be recorded. The writer feels that inadequate efforts have been made in this direction, although he recognizes the difficulties. For instance, in one case, an engineer had small piles of brickbats placed on the beach at about mid-tide, only to find later that children and picnic parties had used them to build picnic fireplaces and toy castles on the beach beyond reach of the waves. Notwithstanding this discouraging experience, it is still believed that some modified application of this procedure is practicable. Pea coal or any obtainable sand and gravel clearly distinguishable from the local beach material would be satisfactory.

One pertinent factor not mentioned by the author, which has much to do with beach maintenance in regions where detritus from the upland constitutes the major source of beach material, is the magnitude and frequency of flood flows. It has been noticeable in California that during years of little rainfall complaints of beach erosion become very prevalent, whereas in years following heavy rainfall and floods shore communities are wont to boast of their wonderful beaches.

Flood control measures which include long jetties at the river mouths, designed to stabilize and maintain these outlets by delivering the flood-borne sand into deep water, are likely thereby to result in beach starvation by depositing the major part of the sand beyond the reach of ordinary wave action instead of near the shore line as in a state of nature. Similarly, upland debris control measures, although most meritorious in preventing land erosion, must inevitably deprive the beaches of much of their natural replenishment. As a consequence, engineers must look forward to an increasing demand for beach protection.

In all these preliminary investigations, it is believed that the chief danger lies in attempting, or being forced, to draw conclusions from data which are necessarily, or otherwise, inadequate. This inadequacy may be the fault of the engineer through his lack of experience; or it may be beyond his control, due to lack of available funds or non-existent historical records.

The effort of an individual to protect his property from marine inroads is not unlike that of a property owner on a river to protect himself from flood damage. Not only is it far more expensive if done individually than if done in cooperation, but it is far less effective and sometimes almost hopeless. Just as flood control, to be effective, must be designed with the entire drainage basin

in mind, so shore protection, to be most effective and economical, must be designed with due thought for the influence from and upon a very considerable extent of shore line. Wherever feasible the coast line should be divided into such sections or stretches as can be considered independently of each other. Then for each section a protection system, such as a "groin field," should be planned before undertaking the construction of any individual part.

The only adequate solution of a beach improvement problem lies in such "coastal planning" considering a stretch of beach of sufficient magnitude to be treated as an independent and complete unit. By thus envisaging a future comprehensive system, the engineer can lay out the most suitable ultimate location for high-water mark and determine the length, number, and spacing of groins which will best produce this result. Parts of this system can then be constructed from time to time with the assurance of an ultimate co-ordinated whole. However, if each property owner is concerned only with the protection or improvement of his own property, he will devise a plan to fit his own selfish ends, which may (but probably will not) harmonize with a general plan which would be best for the public good. The writer realizes fully that this ideal policy is usually practicable only for the publicly employed engineer, and that the engineer employed by an individual owner can seldom consider aught but the interests of his own client.

Furthermore, in most States, the individual riparian owner is legally limited by the "common enemy" doctrine of law. He may erect structures of only such nature and extent as will protect his own property from inroads of the sea. If he limits himself carefully by this requirement, any consequent damage to his neighbor's property is "damnum absque injuria," or injury without legal wrong. However, if he goes beyond work purely for protection and endeavors to improve his property by widening his beach, then he renders himself legally liable for any injury which may result to his neighbor's property.

When all the property owners join in an improvement or protection district, however, they are then mutually responsible for the work of the district, and moreover, the engineer then has a better chance to so design his work and plan his construction (normally constructing the leeward groin first) as not to cause injury to neighboring property.

Not only is co-operation between beach owners essential to secure the best design of a shore protection project, but it is even more so on the grounds of equity and economics. Very often the cost to an upland owner of providing protection for his property alone is out of proportion to the value of the land, whereas his proportional share of a complete shore protection system would be much more reasonable. Even in the latter case, however, it is often inequitable to expect the upland owners to pay the entire cost of this shore protection. The indirect value to the community of an attractive beach frontage, as well as the direct return to the community in increased taxable value, makes it justifiable for the public to bear some proportion of the total cost.

Much has been written of the unsightliness of groins and the saw-toothed beach line which they produce. It is the high, solid groin which is chiefly responsible for this unwarranted criticism. The remedy lies in low groins, built up gradually as the beach builds up, the portion of the groin shoreward

of ultimate high water line being kept at or below the natural beach level. Examples of this may be seen in Fig. 16, which shows low groins, the inshore ends of which have become buried by natural accretion, and which have produced a smooth shore line.

FIG. 16.—GROINS AT LAS TUNAS BEACH, CALIF., SHOWING INSHORE ENDS BURIED BY NATURAL ACCRETION AND RESULTANT SMOOTH SHORE LINE

It is also well known among engineers experienced in shore protection that the ideal method of developing a smooth beach is to keep the groins only a few feet above the beach level, and if possible to raise, gradually, the part of the groins seaward of high-water mark as the beach builds up. This ideal is not always feasible, but the writer has used a very effective modification of this system where the groin is of steel sheet piling. Every fourth pile is driven a few feet lower than the remainder, thus leaving "windows" in the groin. At first these "windows" permit the upper part of the wave (which, of course, is carrying the least sand) to pass on through the groin, thereby decreasing the volume of return flow down the beach slope parallel to the groin and increasing the tendency to sand deposit. Later, when the beach has built up to the bottom of the "windows," the sand passes on through them to the leeward side, thereby fostering a more smooth and uniform beach outline. Should it be desired to build the beach level still higher, these "windows" can be closed by short pieces of sheet pile. The beach then builds up to the top of the groin and buries it to a point seaward of the break in the profile. Fig. 17 shows such a groin under construction, several of which have been built by the State of California, and which have functioned very satisfactorily. The shoreward end of this groin was constructed of wood sheet piling for economy, as it later became buried by natural accretions.

It is understood that on the Atlantic Coast certain groins have been constructed with these "windows" extending out to the seaward end of the groin

and having the bottoms of the "windows" approximately at beach level. It is believed preferable to keep the seaward end of the groin solid, in order that it may tend to divert the littoral drift shoreward where it will be either deposited on the windward side of the groin or pass through the "windows" and be de-

Fig. 17.—Steel Sheet Pile Groin, with "Windows," Under Construction Near Santa Monica, Calif.

posited on the leeward side. Every effort must be directed to minimize the amount of sand being diverted seaward around the outer end of the groin where much of it may be lost.

The author touches only lightly on materials of construction, implying that groins will be built of sheet piling. This type of construction is usually the simplest, and steel is ordinarily preferable to wood, unless the cost differential is too great. Steel is particularly suitable where the beach is underlain by material that is difficult to penetrate. Indeed, steel master piles are often preferable to the round wooden piles indicated in Fig. 11.

A strong argument against wood piling is its buoyancy. The writer has seen wood groins fail by becoming loosened in a storm and then "popping" out, without being broken off. Perhaps the penetration was inadequate in these cases, but steel piling would have required less penetration for safety. Sand erosion of the steel is more of a problem than corrosion. Asphalt and sand coating has been found more effective protection than most of the commercial paint coverings.

Sometimes it becomes necessary to construct groins where no type of piling is practicable. Of course, the rock mound type familiar on the New Jersey

FIG. 18.—CONCRETE BLOCK GROINS, SANTA BARBARA COUNTY, CALIFORNIA

coast can be used where the groin (or "jetty," as it is called in that locality) is large enough to justify use of rock sizes capable of withstanding wave action; but such structures are likely to be lacking in sand-tightness. Solid concrete has been used successfully in Massachusetts and California, and possibly elsewhere, where there is no danger of settlement.

Fig. 19.—Concrete Block Groin, Santa Barbara, Calif. Inshore End Raised by Monolithic Cap

In certain instances the writer has combined the advantages of the two types by using articulated massive concrete blocks. This construction produces a groin that is sufficiently massive to withstand storms and yet which can settle irregularly without failure. Fig. 18 shows groins of this type—Fig. 18(*a*) being a view during construction, Fig. 18(*b*) soon after their completion, Fig. 18(*c*) the subsequent accretion, and Fig. 18(*d*) a view of a groin under storm conditions. If it is desirable to raise the groin after it has reached its ultimate settlement, a concrete cap can be constructed on top of the blocks which, because of their shape, provide an effective bond between the old and the new work. Fig. 19 shows a groin of this type, the shoreward part of which has been capped.

The writer agrees with the conclusions of the author that: (*a*) The ratio of groin length to groin interval should be between 1 : 1 and 1 : 3; (*b*) spacing closer than 1 : 1 is non-injurious but is uneconomical; and (*c*) spacing greater than 1 : 3 is usually ineffective.

This raises the question, however, as to what is the effective measure of groin length. Is it the actual constructed length, measured from the bulkhead regardless of its location, to the seaward extremity of the groin, or is it the length of groin remaining exposed above the beach slope after the groin has

accomplished its major purpose in building up the beach? It appears rather obvious to the writer that the second is the logical conclusion because, after the beach has reached a condition of equilibrium subsequent to building of the groin, one certainly cannot consider the buried part of the groin as having any influence on wave action.

As a further rule for groin spacing the author advocates, subject to the foregoing limits, that the groins should be so spaced that a line drawn through the seaward end of one groin parallel to the direction of wave approach will intersect the beach at the root of the next groin. This sounds simple, but what is "the direction of wave approach"? Is it that in deep water or is it that near the plunge point? On approaching near enough to shore to "feel the bottom," all waves gradually change their direction so that, when they break, their fronts are approximately parallel to the shore. Therefore, it is essential to know where to observe the direction of approach. A line through the seaward ends of the groins would appear to be the logical point of observation, for that is where the waves are intercepted.

Furthermore, which kind of wave shall be used as a criterion, the "normal" waves, the direction of which is most predominant through the year, or the storm waves, which, although less frequent, are much more powerful? In Southern California the fair weather waves approach generally from the southwest or west and predominate through two-thirds to three-fourths of the year, whereas the southeasterly or southerly storm waves occur only in the three or four winter months. There is little question here in adopting the prevailing fair weather direction. In other localities, however, the problem may be less simple, for the wind and wave direction may be much more variable.

The author's rule for groin spacing, based on direction of wave approach, does not seem entirely clear. He states that "In the case of storm action parallel to the coast the resulting waves do not travel parallel, but the shore ends are retarded so that the wave reaches the shore at an angle of about 16 degrees." Applying the author's own rule to this case would appear to justify a groin interval equal to only approximately one-fourth the groin length, if the angle is measured from the perpendicular, or four times the groin length, if the angle is measured from the shore. It is hoped that the author will amplify his rule.

In California the State Division of Lands is gradually accumulating a record of seacoast structures within the State, their behavior, and their effect on the coast line. Since 1931 State permits have been required for construction and maintenance of all structures extending seaward across the State-owned tide and submerged lands. Each applicant is required to submit for approval plans of the proposed structure, a map showing the present (and past, so far as available) lines of low tide, high tide, and extreme reach of the sea in the vicinity, accompanied by photographs. From time to time after completion of a structure, inspections are made and photographs obtained to record the effect produced by the structure on the shore line. By this procedure it is planned to accumulate a reservoir of information that should be of service, not only to the State, but to the engineering profession in future shore protection and development.

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DISCUSSIONS

POLLUTION OF BOSTON HARBOR

Discussion

By Harrison P. Eddy, Jr., and Samuel A. Greeley, Members, Am. Soc. C. E.

Harrison P. Eddy, Jr.,[10] and Samuel A. Greeley,[11] Members, Am. Soc. C. E. (by letter).[11a]—An excellent description of one of the earliest comprehensive sewerage systems, and of conditions in Boston Harbor after these works had been in service many years, is presented in this paper. The authors are to be complimented on the completeness and thoroughness of their work.

This area has been most fortunate in having available favorable tidal currents. Although the photographs in the paper give the appearance of an extremely unclean condition on the surface of the harbor waters from the Moon Head outlet, the intermittent discharge and the prompt removal and dispersion of the sewage by the currents have made possible the operation of this system for about fifty years without creating intolerable conditions. At the Nut Island outlet, the submergence of the outlets tends to offset the continuous discharge; and at Deer Island submergence, strong tidal currents, and proximity to the ocean are factors that have helped to reduce the unfavorable effects of pollution.

Under the provisions of a resolve of the Massachusetts Legislature of 1938, a special commission was appointed to make an investigation relative to the systems of sewerage and sewage disposal in metropolitan Boston. As a part of its investigation, this commission retained the writers' respective offices as consulting engineers to investigate and report upon the following two major subjects:

(*a*) Pollution in the Mystic River, and its major tributaries, and in the Charles River, and methods of correction; and

(*b*) Proposed methods of sewage treatment for the main outlets in Boston

Note.—This paper by Arthur D. Weston and Gail P. Edwards, Members, Am. Soc. C. E., was published in March, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1939, by E. Sherman Chase, M. Am. Soc. C. E.; and December, 1939, by Messrs. F. E. DeMartini, and A. M. Rawn.

[10] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[11] (Greeley & Hansen), Chicago, Ill.

[11a] Received by the Secretary January 2, 1940.

Harbor of the North and South Metropolitan Sewerage Districts and the Boston Main Drainage System.

In making their investigation and report, the engineers reviewed House Document No. 1600, December, 1936,[3] the testimony gathered by the special commission, and many other documents, and made personal observations of metropolitan Boston and the harbor, and of the main structures of the sewerage systems. Conclusions pertinent to the paper under discussion are as follows:

Mystic and Charles Rivers.—Objectionable conditions exist, at times, in the Mystic River, and its tributaries, and in the Charles River, due to pollution from sewage. This pollution occurs because, in times of storm, the capacity of the existing intercepting sewers is insufficient to carry away the sewage and storm water that are delivered to them, and the excess beyond their capacity is discharged into the streams through overflow channels provided for this purpose.

Correction of the objectionable conditions in the Mystic River, and its tributaries, and in the Charles River can be obtained best and most economically by the construction of storm overflow conduits of sufficient capacity so that no overflows into the streams will occur until the volume of storm water flowing in the sewers has provided sufficiently great dilution of the domestic sewage to prevent objectionable conditions from such overflows. If these recommendations are followed, it should no longer be necessary for Boston and a few other municipalities to separate storm water and sewage in areas tributary to the Charles River, as now required, when new sewers are built or old sewers are replaced.

The existing Mystic Valley sewers were relieved by the construction, in 1936 and 1937, of the North Metropolitan relief sewer from Reading, Mass., to a point a short distance below Craddock Dam in Medford, Mass. This relief sewer is now (1940) being extended to the East Boston pumping station; and it is proposed by the consulting engineers that the North Metropolitan relief sewer be extended from East Boston to Deer Island. The estimated cost of construction of the storm overflow conduits, pumping stations, and force mains recommended, including the extension of the North Metropolitan relief sewer to Deer Island, is $14 695 000.

Pollution of Boston Harbor.—To aid them in reaching a conclusion as to the limiting effect of sewage in Boston Harbor upon the use of the waters, and as to the need for sewage treatment, the consulting engineers have adopted a standard for the desirable condition of the harbor waters, after consideration of standards proposed and adopted elsewhere and of the existing sanitary conditions. They have assumed that, for the bathing beach waters of the harbor, a single *B. coli* index of 1 000 per 100 cc is a warning that excessive contamination may be present at times, and that an average index in excess of 3 000 *B. coli* per 100 cc shows that the waters may be dangerous to the public health.

In examining the bacteriological analyses made by the Massachusetts Department of Public Health during the investigations of Boston Harbor in

[3] See "Report of the Special Commission on the Investigation of the Discharge of Sewage into Boston Harbor and Its Tributaries," House Doc. No. 1600, December, 1936.

1929–1930, and 1935–1936, the consulting engineers made a large number of charts showing the distribution of the *B. coli* index throughout the harbor. One of these, Fig. 13, has been selected as giving a representative picture of the average or general bacteriological condition of the harbor. It shows the average and maximum *B. coli* indices of all samples collected at each sampling station during the 1935 investigation, as well as the areas where the *B. coli* index was probably in excess of 3 000 per 100 cc and those where there was a probable *B. coli* index between 1 000 and 3 000 per 100 cc.

It is of interest to note that, in general, the number of days on which samples were taken was five to ten at each station during each of the years 1935 and 1936. During 1935, the prevailing winds on the sampling days were from the west and southwest, and during 1936 they were from the east and southwest. The higher *B. coli* indices occurred when the wind was blowing from the north and east. There is a possibility, therefore, that if all samples had been taken on days when the wind was blowing from the north or northeast, the average of the analyses would have shown much higher *B. coli* indices for the shore waters.

The consulting engineers have come to the conclusion that the bacterial content of the shore waters of the harbor, as evidenced by the *B. coli* index, in some places is at times higher than what is generally considered a reasonable upper limit for the safety of bathing beach waters. Furthermore, the proximity to the shore, of harbor waters that are highly contaminated with raw sewage from the outlets, is a potential source of excessive bacterial pollution which is certain to increase in the future. In addition, the stirring up of the mud at the bottom of the harbor, which was found to contain many more bacteria characteristic of pollution than did the water at the surface, by strong onshore winds, is a potential source of excessive bacterial pollution. Present (1940) conditions are potentially dangerous to the public health to such an extent that corrective and preventive measures should be undertaken as soon as practicable by the construction of sewage treatment works.

Sewage Treatment Plants.—The proposed method of sewage treatment is essentially the same at each of the three main outlets, and comprises the following principal elements: Fine racks, grit chambers, aero-chlorinating tanks, sedimentation tanks, sludge storage tanks, equipment for disposal of sludge at sea, and chlorination equipment.

It is contemplated that the sedimentation tanks will be equipped with mechanical means for removal of sludge, grease, and floating solids. In order to increase the proportion of grease that can be removed by skimming, it is deemed advisable to aerate the sewage and apply a small quantity of chlorine to it. It is planned to burn the scum in suitable incinerators. The projects include the disposal of sludge by barging to sea, where it may be discharged without causing complaint. The excessive bacterial pollution of harbor and shore waters during the season of recreation can be prevented by disinfection of the sewage with liquid chlorine.

The total estimated costs of construction of the three proposed plants, with capacities estimated for 1955, together with the estimated population and the annual average quantity of sewage tributary to each of them in that year, are given in Columns (2), (3), and (4), Table 13. On the assumption that

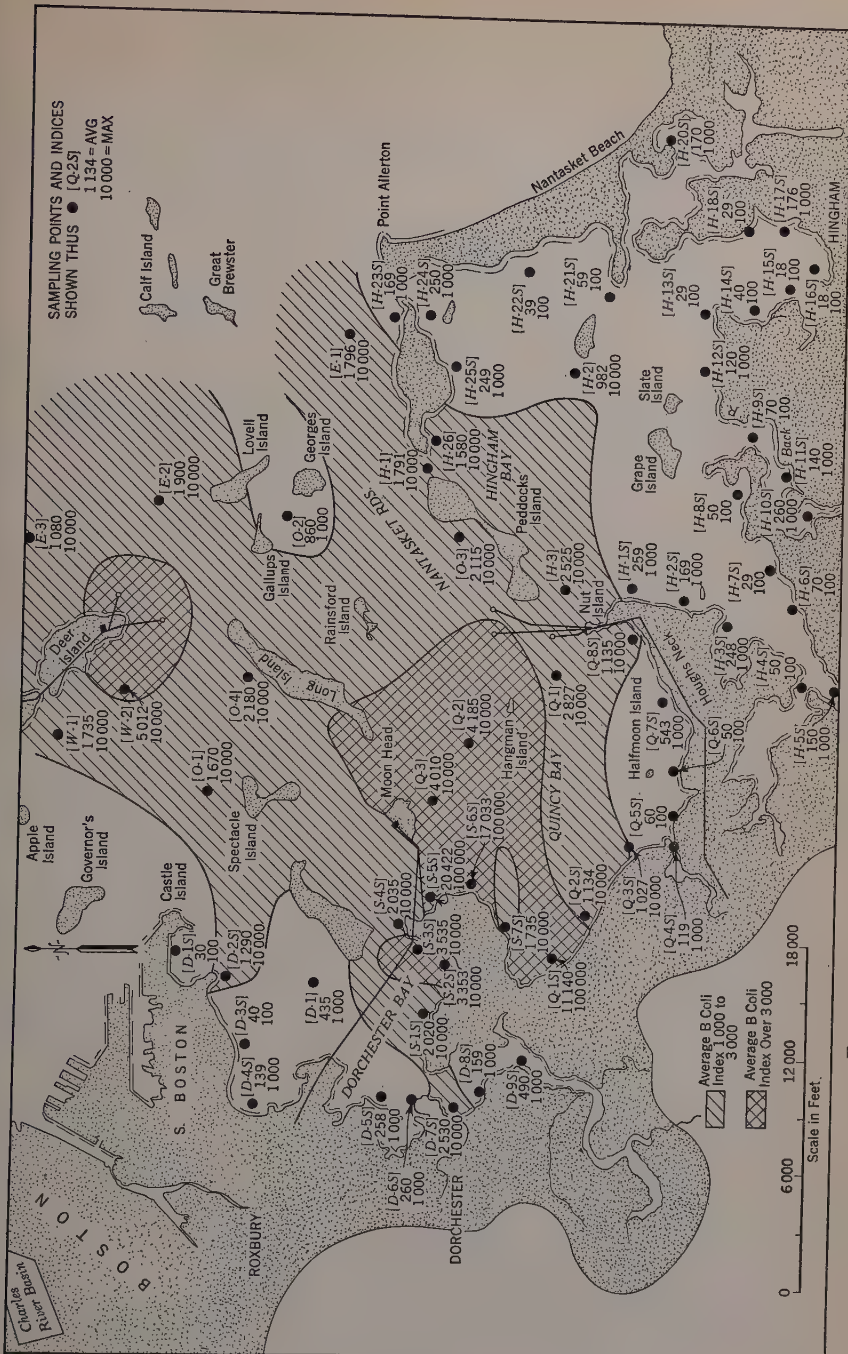

FIG. 13.—AVERAGE AND MAXIMUM *B. COLI* INDICES OF ALL SAMPLES; 1935 SERIES

20-yr, $2\frac{1}{4}\%$ serial bonds of equal annual retirement payments are issued in 1941, the total annual costs of the three projects, as of 1943, are estimated to be as shown in Column (7), Table 13.

TABLE 13.—Costs of Construction, with Capacities Estimated for 1955

Outlet	Estimated population in 1955	Average quantity of sewage, in million gallons daily	Estimated Costs, in Dollars			
			Construction	Annual Costs		
				Operation	Capital charges	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Boston Main Drainage....	400 000	85.0	3 180 000	240 000	223 400	463 400
North Metropolitan......	750 000	125.0	3 784 000	284 000	265 800	549 800
South Metropolitan.......	900 000	112.5	3 058 000	249 000	214 400	463 400
Total...............	2 050 000	322.5	10 022 000	773 000	703 600	1 476 600

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE UNIT HYDROGRAPH PRINCIPLE APPLIED TO SMALL WATER-SHEDS

Discussion

BY MESSRS. LEROY K. SHERMAN AND RAPHAEL G. KAZMANN

LEROY K. SHERMAN,[10] M. AM. SOC. C. E. (by letter).[10a]—A careful analysis of unit hydrographs, derived from good rainfall and run-off data, has been made by the author. He has demonstrated conclusively that the unit graph method is applicable to small areas. In his earlier work with the unit graph, the writer dealt with 24-hr total rainfall records. Consistent graphs, which neglected rain intensity and distribution, naturally could not be found unless the area was large enough to furnish time of transit and channel storage sufficient to "iron out" the effects of changing rain patterns. When recording gages are used, as in the author's examples, consistent results will be found on both large and small areas. It has been found that:

(*a*) The unit of time must be less than, and preferably a fraction of, any concentration period; and

(*b*) The unit graph, or the percentage distribution graph, on the larger areas, reflects the pattern of effective rainfall. For example: A storm centered on the lower end of the basin gives a relatively high and early peak, and conversely, a storm centered at the upper part of the basin gives a lower peak rate of run-off for the unit graph. Both are correct. In applications for run-off estimates, the type of graph is selected which is best suited to the given rainfall pattern. This gives better results than the average of several derived unit graphs.

On the derivation of base flow, the author's reasoning, for the adoption of the upper line "*b*," seems logical for the particular basins involved. It is assumed that the author has located Point 4, on the tail of the hydrograph, by actual field observation. That is, Point 4 is the observed time when all material overland flow into the stream has ceased. The writer is of the opinion that many base flow lines are drawn for flood hydrographs which show too great an increase in base flow during the period of surface run-off. Many cases have

NOTE.—This paper by E. F. Brater, Jun. Am. Soc. C. E., was published in September, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1940, by Franklin F. Snyder, Jun. Am. Soc. C. E.

[10] Cons. Engr., Chicago, Ill.

[10a] Received by the Secretary December 22, 1939.

been found in which the base flow was actually negative. This is due to bank storage of water by infiltration at overbank or high stages of the stream. Before ground-water outflow can increase under high river stages, the infiltration or bank storage must become sufficient to raise the ground-water table. The correct method of solving the problem on an experimental area such as this is, as the author suggests, to have observation wells placed along the bank of the stream for measurement of the ground-water outflow profile.

Mr. Brater has made a careful study of run-off coefficients. This study would have been more useful had he applied his work to infiltration capacity instead of run-off coefficients. Run-off has no definite relation to rainfall, since it is rainfall minus infiltration and other losses. Considerable study and investigation have been made on infiltration capacities and loss rates.[11] The time has come already when such loss rates can, and are, being used in engineering practice for estimating run-off from rainfall. The use of these infiltration capacities or loss rates gives far more accurate results than can be obtained by the most qualified hydrologists in estimating, or guessing, at the coefficient of run-off. There is no run-off unless the rainfall has an intensity greater than the infiltration capacity of the soil. The writer considers it a fundamental mistake to "leave intensity out of the picture." The author is adhering to an old procedure which is fundamentally erroneous and rapidly becoming obsolete.

As an illustration of the infiltration method, Table 5 has been compiled from

TABLE 5.—AVERAGE INFILTRATION CAPACITIES

Basin	Date of storm	Average infiltration capacity, in inches per hour	Duration of effective rain, in minutes	Reference
Coweeta 7a	6/12/36	2.35−	25	Fig. 2 (*b*)
Coweeta 7b	6/12/36	2.35+	25	Fig. 2
Coweeta 7	4/ 1/36	2.76	10	Fig. 8 (*b*)
Coweeta 7	9/29/36	2.10	40	Fig. 8 (*a*)

data furnished by the author. The average infiltration capacity in this table was computed by the direct method, which is premised on the fact that the volume of rain excess is practically equal to the volume of observed surface run-off. Note the consistent decrease in infiltration capacity with duration of effective rainfall. The absorptive capacity of the initially dry soil is greater than the more saturated soil. The average infiltration capacity, following excess rain of more than one or two hours duration, will be only a little greater than the ultimate infiltration capacity.

When applying ultimate infiltration capacity to given rainfall, for the purpose of deriving the volume of run-off, it is frequently desirable to know the initial loss rates. The writer is of the opinion that the author's procedure of utilizing the pluviagraph (Fig. 8) to determine variation in run-off percentages can also be used to determine the changes in initial loss rates.

It is pertinent to note that the forested basins used by the author represent an extreme limit of low run-off and high infiltration capacity. This is illus-

[11] "Surface Runoff Phenomena," by R. E. Horton, M. Am. Soc. C. E., *Bulletin* 101, Voorheesville, N. Y.; also *Transactions*, Am. Geophysical Union, 1936, p. 302; 1937, p. 371; and 1938, pp. 430–436.

trated by the following derived ultimate infiltration capacities:

Place	Ultimate infiltration capacities, in inches per hour
Bethany, Mo.	0.15
Illinois	0.10–0.30
Industry, Ohio	0.16
Red River Basin, Okla.	0.21–0.25
North Concho Basin, Tex.	0.30
Lincoln, Nebr.	1.00
Colorado Springs, Colo.	2.00
Coweeta 7, N. C.	2.10
Copper Basin, No. 1, N. C.	3.00 ±

The flat lines of "mass surface run-off" in Fig. 8 indicate the high rate of infiltration on Coweeta No. 7. The writer has replotted the mass run-off for the storm of September 29, 1936, with an expanded vertical scale. This graph shows clearly the relation of run-off to the intensities of rain in excess of infiltration capacity.

RAPHAEL G. KAZMANN,[12] ESQ. (by letter).[12a]—A secondary conclusion advanced in this interesting paper is worthy of some attention. After describing two consecutive rains which followed one another closely, Mr. Brater states (heading, "Applications of the Distribution Graph: Run-Off Coefficients and Infiltration Capacity"), "The conclusion may be drawn, then, that the infiltration capacity increased considerably in the space of 1 hr despite the 1.60 in. of rainfall immediately preceding." This conclusion was based on the data from Bent Creek 3 which indicated that the run-off from Rain No. 4 was about one-half the run-off which resulted from Rain No. 3, although the two rains were "nearly equal in amount and intensity." The question may be raised logically: Can the infiltration capacity of a water-shed be increased by saturating it?

Admittedly, circumstances are conceivable under which such a phenomenon might occur: For example, the presence of a top layer of frozen earth that had been thawed out by the first rain might bring an apparent increase in the infiltration capacity of a water-shed. The same result would occur if the soil were very dry with all the interstices filled with air, if the first rain were a deluge and the second a slower, steadier precipitation. The circumstances would be analogous to those of an air-locked filter. In general, however, experience seems to indicate that the infiltration capacity of a water-shed decreases as the ground becomes more saturated, finally reaching a more or less well-defined minimum.

It is possible, of course, that during the period upon which the author based his conclusion the upper layer of soil was in a frozen or semi-frozen state. If, on the other hand, there was no frost in the ground (April is an in-between month, anyway), there is another possibility implicit in the data collecting methods used in the observations. It lies in the failure to correlate

[12] Graduate Asst. in Civ. Eng., The Pennsylvania State College, State College, Pa.

[12a] Received by the Secretary January 15, 1940.

rainfall intensity with the other observations. Mr. Brater states (heading, "Applications of the Distribution Graph: Run-Off Coefficients and Infiltration Capacity"):

> "The correlations attempted in the following pages are based on quantity of rainfall and do not specifically take rainfall intensity into account. Although it is recognized that intensity of rainfall has an important bearing on the percentage that becomes surface run-off, the intense rains are usually large ones, and therefore, if the run-off coefficient or the infiltration capacity is correlated with quantity of rainfall, the intensity will be taken into account somewhat."

Considering this statement, it is not difficult to surmise how a questionable conclusion concerning increased infiltration capacity might have been deduced. From the rainfall records the quantity of rain and the total time of each of the periods of precipitation was known; but the distribution of the precipitation during these periods was unknown. This was due to the fact that Bent Creek 3 is the smallest of the Bent Creek water-sheds; consequently, since there were not enough recording rain gages to go around (five recording gages *versus* eight water-sheds), it probably did not receive a recording gage.

Probably the first rain was "spotty." Periods of light precipitation alternated irregularly with periods of very intense rainfall. The rain which followed (Rain No. 4) was probably steadier and had fewer periods of intense precipitation. Without recording gages, unless an observer was on the water-shed during the rains, this "fact" would be unknown. By simply dividing the total quantities of precipitation by the periods of time involved, it would appear that the intensities of precipitation were the same, whereas, although average intensities were nearly alike, the actual intensities differed widely. Assuming that the picture sketched is correct, the phenomenon was observed correctly but an incorrect, possibly too hasty, conclusion was drawn. The run-off from the second storm was less, chiefly because the rain, in general, was more evenly distributed and did not come down in spasmodic torrents. This more gradual rainfall gave the soil (eroded and wet though it was) a better chance to accommodate the rain—that is, the rainfall could infiltrate rather than run off on the surface.

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DISCUSSIONS

FIELD TESTS OF A SHALE FOUNDATION

Discussion

BY JACOB FELD, M. AM. SOC. C. E.

JACOB FELD,[5] M. AM. SOC. C. E. (by letter).[5a]—A complete and unbiased description of field and laboratory tests, carefully planned and executed, is given in this paper. The value of the large-scale field tests is without question; the author's admitted lack of success in making the laboratory test results agree with the field test results should be considered a marked contribution to the study of foundations. No more valuable warning can be cited of the dangers from indiscriminate extrapolation of small-scale laboratory tests for use in design, unless such tests are tempered by actual experience with similar foundation materials or superseded by large-scale field tests.

As Mr. Niederhoff explains, the shale seemed to follow the laws of elasticity up to certain unit loadings, but with definite plastic flow characteristics.

A possible explanation can be given of the inconsistencies of the settlement curves in Fig. 8 if a limit is set to the distance beyond the loaded areas through which resistance to settlement is built up by the internal strain in the shale. Assuming that the load-carrying area below the loaded surface remains square (for simplicity of explanation) and that the characteristics of the shale were the same in each of the three loaded plate tests, the loaded areas in each test at the

TABLE 3.—AVERAGE LOADS CARRIED PER UNIT AREA

Test plate	Total load, in tons	Area, in square feet, at depth d	Avenue load, in tons per square foot
6 in. by 6 in.	2.5	2.5 by 2.5 or 6.25	0.4
12 in. by 12 in.	10	3 by 3 or 9	1.1
24 in. by 24 in.	40	4 by 4 or 16	2.5

same depths are not stressed uniformly. For a depth where the loaded areas have increased in dimension by 1 ft on each side of the test plate, the average loads carried per unit area are as given in Table 3. These data are given only

NOTE.—This paper by August E. Niederhoff, Assoc. M. Am. Soc. C. E., was published in September, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1940, by Harry H. Hatch, M. Am. Soc. C. E.

[5] Cons. Engr., New York, N. Y.

[5a] Received by the Secretary January 10, 1940.

to indicate the qualitative effect; and a true value can be obtained only by the application of the Boussinesq formula if the proper Poisson ratio and parameter are known. However, since the total settlement is the accumulation of strains, not only directly below the loaded plate but also in the body of the shale to such depths as appreciable additions to the stress conditions in the shale result from the loading, the comparative data illustrate the result to be expected. In the 6-in. by 6-in. plate, the average stress in the shale decreased rapidly with depth; and the settlement was small and reached a maximum in about 100 days. The permanent set is an indication of the amount of load transferred to adjacent shale volumes and resisted by such compression, the material having been changed to a denser, and better load-carrying shale. In the 12-in. by 12-in. plate, the average stress at that depth was two and three fourths times as great, the total settlement was correspondingly greater, and the adjacent shale, after 140 days, had not yet taken up the load by being compressed. In the 24-in. by 24-in. plate, the same condition was also true to a larger degree.

An attempt to apply the results of a 6-in. by 6-in. test plate to the design of a dam foundation under the same unit loading can result only in unforeseen and apparently unexplainable settlements.

The description and results of the large-scale shear tests in shales are especially valuable as an addition to the compilation of data on foundation phenomena.

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DISCUSSIONS

STRESS DISTRIBUTION AROUND A TUNNEL

Discussion

By J. H. A. Brahtz, Esq.

J. H. A. Brahtz,[25] Esq. (by letter).[25a]—Congratulations are due the author for his excellent presentation of a very difficult theoretical subject. It is highly gratifying to have studies of this caliber brought to the attention of the engineering profession. In the mathematical theory of elasticity, relatively few exact solutions exist that may be applied directly to engineering problems. This paper is a definite contribution to this small group of exact solutions.

A characteristic behavior of additions to the exact-solution group seems to be that the underlying mathematics become more abstruse and complicated with each addition. This is particularly true in the cases treated by Mr. Mindlin, but in no way detracts from the value of the paper. However, it does limit its broad use and application to a relatively small group of engineers with a highly specialized mathematical background. Apparently, the author is well aware of this limitation and has minimized the attendant difficulties by presenting, in tabular and graphical form, the results of many computations. These simplify and reduce the labor greatly in applying the theoretical results to engineering problems.

In his study of the stresses due to body forces, Mr. Mindlin has considered three possible conditions that differ only in the ratio of horizontal stress to vertical stress along the vertical boundaries. To the mathematician the choice of this ratio represents no problem; it is merely called a boundary condition and is assumed to be known. To the practicing engineer the proper choice of this ratio is of utmost importance. For lack of definite information on the proper value of this ratio, the engineer is likely to employ a value that produces the most critical state of stress. This is on the side of safety but scarcely ever on the side of economy. Although the author has left this value "floating" (as he should, since a knowledge of it comes under the science of soil mechanics or geophysics, and not mathematics), it seems important to stress the need of experimental and statistical knowledge on this phase of foundation engineering.

Note.—This paper by Raymond D. Mindlin, Assoc. M. Am. Soc. C. E., was published in April, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1939, by Messrs. W. O. Richmond, and Jacob Feld.

[25] Director, Photoelastic Laboratory, U. S. Dept. of the Interior, Bureau of Reclamation, Denver, Colo.

[25a] Received by the Secretary January 2, 1940.

It would have been interesting if the author had included one or more photoelastic checks and had compared the experimental maximum shear stresses with the computed values. It is realized, however, that body forces are difficult to apply to photoelastic models and that even when M. A. Biot's[26] method of employing boundary forces is used, formal and practical difficulties arise in the application of a continuously varying normal load on the closed boundary that corresponds to the tunnel surface.

Mr. Mindlin confined the application of the bipolar theory to body force or "dead load" stresses. For structures having shapes similar to those given in the paper, other common conditions of loading exist. These include the cases in which either the straight or circular boundaries are subjected to uniformly distributed loads. Although the general form for these cases has been given elsewhere,[27] it is felt that the particular cases in which one boundary is a straight line should be included in the paper, for completeness. Loadings of this type frequently arise in engineering practice. Usually they occur simultaneously with the case treated by the author. An example of this condition would be a circular tunnel in bed-rock below a river-bed or earth embankment. Sometimes they occur as the only loading; for example, a vertical shaft near the up-stream face of a masonry dam or a pressure pipe, or conduit, near a free surface. The latter is the case of uniform normal pressure on the inner boundary.

The writer is presenting the results obtained by auxiliary studies covering the conditions of uniform normal loads along the boundaries. The first part of the study is purely mathematical and is based upon Jeffery's general theory for stresses in a circular plate having an eccentric hole. A uniform normal pressure is applied to the outer boundary. If the radius of this boundary is assumed to increase without limit, the segment of the outer boundary in the vicinity of the hole approaches the limiting condition of a straight line. It should be noted that, in this limiting process, the outer boundary is still loaded on all sides, and the stress formulas obtained for the limiting case must be for the condition of the infinite boundaries, as well as the straight boundary, being loaded uniformly.

Although this is an important loading condition, a more common condition is one in which the straight boundary is loaded fully, and the vertical infinite boundaries are loaded only partly. The stresses for this condition can be obtained, however, by combining the results of a mathematical solution with those of photoelastic experiments, or by combining two mathematical solutions. The two mathematical solutions to be combined are the one given by the writer in this discussion and the solution by Mr. Jeffery referred to by the author.[11] However, Mr. Mindlin has called attention to an inconsistency in Mr. Jeffery's solution, and, rather than attempt to correct this fundamental error, the writer has used a combination of experiment and theory. The experimental problem treated by the writer is the problem treated by Mr.

[26] *Journal of Applied Mechanics*, June, 1935.

[27] See, for example, "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 313, *et seq.*

[11] "Plane Stress and Plane Strain in Bipolar Coordinates," by G. B. Jeffery, *Transactions* of the Royal Society, London, England, Series A, Vol. 221 (1920), pp. 265–293.

Jeffery. A table of numerical values for stresses at critical points has been developed by S. Timoshenko.[28] For the smallest value of $\frac{d}{r}$ given in the table, the boundary stress on the straight boundary opposite the hole is approximately twice the average stress in the plate, and of opposite sign. In his experimental study the writer was unable to note any change of sign in the stress along this boundary. An inspection of the isochromatic photographs included herewith bears out this point. Finally, the stresses are given for the cases in which a uniform pressure exists only on the internal boundary.

Another interesting application of bipolar co-ordinates arises in connection with the problem of percolation into a tunnel in pervious foundation or a shaft of circular cross-section situated near the up-stream face of a masonry dam.

Uniform Normal Loads on Boundaries.—Again making use of the bipolar transformation, shown in Fig. 1, the following boundary conditions are specified:

(I) The boundary $\alpha = \alpha_1$ is subjected to a normal load $\sigma_\alpha = 1$ and is free of shear—that is, $\tau_{\alpha\beta} = 0$.

(II) The boundary $\alpha = \alpha_2$ is free of load—that is, $\sigma_\alpha = \tau_{\alpha\beta} = 0$; and α_1 and α_2 are any of the positive α-circles shown in Fig. 1.

Now consider the function (see Equation (30))—

$$\frac{x_6}{J} = F = \sum_{n=0}^{\infty} f_n(\alpha) \cos n\,\beta \qquad (71)$$

The first two functions ($n = 0$ and $n = 1$) are

$$F_0 = A_0 \cosh \alpha + B_0\, \alpha \cosh \alpha + C_0 \sinh \alpha + D_0\, \alpha \sinh \alpha \qquad (72a)$$

and

$$F_1 = (A_1 \cosh 2\,\alpha + B_1 + C_1 \sinh 2\,\alpha + D_1\, \alpha) \cos \beta \qquad (72b)$$

The relations, $D_0 = 0$ and $-D_1 = B_0$, have been shown by the author to exist in order that displacements may be single valued; and he has shown that A_0 may be placed equal to zero since $B_1 \cos \beta$ gives the same type of stress. Hence $F_0 = -D_1\, \alpha \cosh \alpha$ and

$$F = F_0 - F_1 = (A \cosh 2\,\alpha + B + C \sinh 2\,\alpha) \cos \beta + D\,\alpha\,(\cos \beta - \cosh \alpha) \qquad (73)$$

Substitution of Equation (73) in the general stress equations (Equations (19)) gives

$$a\,\sigma_\alpha = A\,(\cosh 2\,\alpha - 2 \sinh 2\,\alpha \sinh \alpha \cos \beta) + B + C\,(\sinh 2\,\alpha - 2 \cosh 2\,\alpha \sinh \alpha \cos \beta) + D \sinh \alpha\,(\cosh \alpha - \cos \beta) \qquad (74a)$$

$$a\,\sigma_\beta = A\,[2 \cosh \alpha \cos \beta - \cosh 2\,\alpha\,(4 \cos^2 \beta - 1 - 2 \cosh \alpha \cos \beta)] + B + C\,[2 \sinh \alpha \cos \beta - \sinh 2\,\alpha\,(4 \cos^2 \beta - 1 - 2 \cosh \alpha \cos \beta)] - D \sinh \alpha\,(\cosh \alpha - \cos \beta) \qquad (74b)$$

and

$$a\,\tau_{\alpha\beta} = (2\,A \sinh 2\,\alpha + 2\,C \cosh 2\,\alpha + D)\,(\cosh \alpha - \cos \beta) \sin \beta \qquad (74c)$$

The constants A, B, C, and D are now determined by the boundary condi-

[28] "Theory of Elasticity," by S. Timoshenko, New York, 1934, p. 79.

tions (I) and (II). Omitting mathematical details, the constants are found to be:

$$A = \frac{a}{2} \frac{\sinh(\alpha_2 + \alpha_1)}{\sinh(\alpha_2 - \alpha_1)\,[1 - \cosh(\alpha_2 + \alpha_1)\cosh(\alpha_2 - \alpha_1)]} \quad \text{(75a)}$$

$$B = A \frac{\sinh(\alpha_2 - \alpha_1) - \sinh 2\,\alpha_2 \cosh(\alpha_2 - \alpha_1)}{\sinh(\alpha_2 + \alpha_1)} \quad \text{(75b)}$$

$$C = -A \coth(\alpha_2 + \alpha_1) \quad \text{(75c)}$$

and

$$D = 2\,A \frac{\cosh(\alpha_2 - \alpha_1)}{\sinh(\alpha_2 + \alpha_1)} \quad \text{(75d)}$$

It has been stated that Equations (75) are applicable to any two circular boundaries, $\alpha = \alpha_1$ and $\alpha = \alpha_2$, as shown in Fig. 1. It gives the stresses for

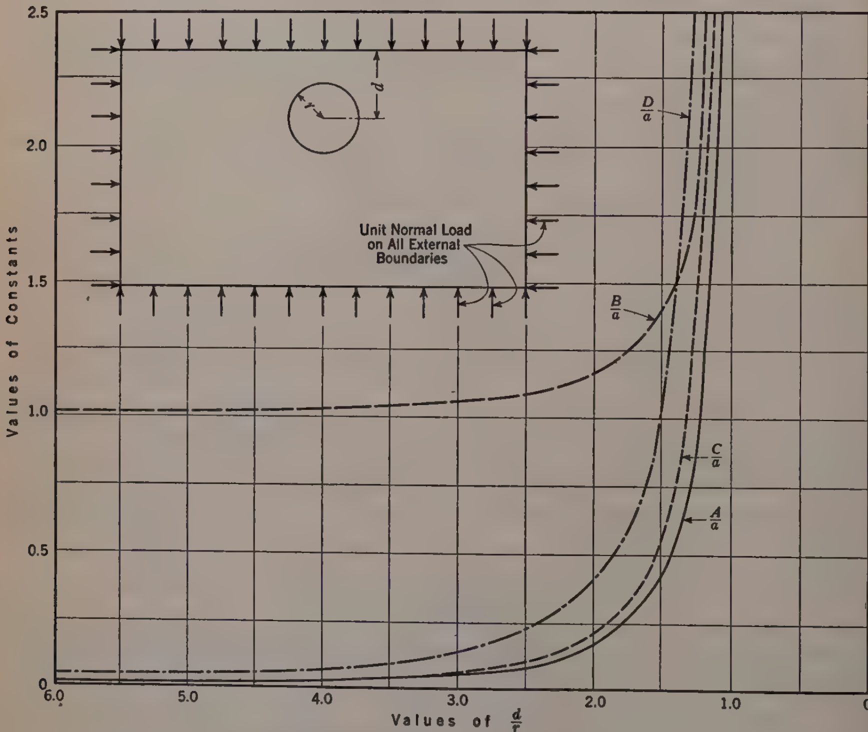


FIG. 8.—CONSTANTS A, B, C, AND D WHEN $\alpha_1 = 0$

unit normal tension on the boundaries of a circular cylinder with an eccentric circular hole.[27] If the outer boundary is free of force, and if a unit compression force acts on the boundary of the hole, it is only necessary to superimpose a unit hydrostatic state of compression, $\sigma_\alpha' = \sigma_\beta' = -1$, $\tau_{\alpha\beta}' = 0$, on to the stresses of the first solution.

Application to a Circular Tunnel.—If the ground surface is assumed plane (that is, a cylinder of infinite radius), α_1 is placed equal to zero in the formulas;

hence, Equations (75) become

$$A = \frac{a}{2\,(1 - \cosh^2 \alpha_2)} \qquad (76a)$$

$$B = A\,(1 - 2 \cosh^2 \alpha_2) \qquad (76b)$$

$$C = -\,A \coth \alpha_2 \qquad (76c)$$

and

$$D = -\,2\,C \qquad (76d)$$

The values of the constants for this case are plotted in Fig. 8 for various ratios of $\frac{d}{r}$, in which d is the depth of the center below the ground surface and r is the radius of the tunnel. The next step is to plot the stresses for unit compressive load, obtained by stress Equations (74), around the hole, along the ground surface, and along a vertical line through the center of the hole for $\frac{d}{r} = 6, 5, 4, 3, 2, 1, 1.5$ and 1.25, respectively.[29] All stresses due to this loading are compression and can be plotted in terms of the unit load. Fig. 9 illustrates the form of curves for $\frac{d}{r} = 1.5$.

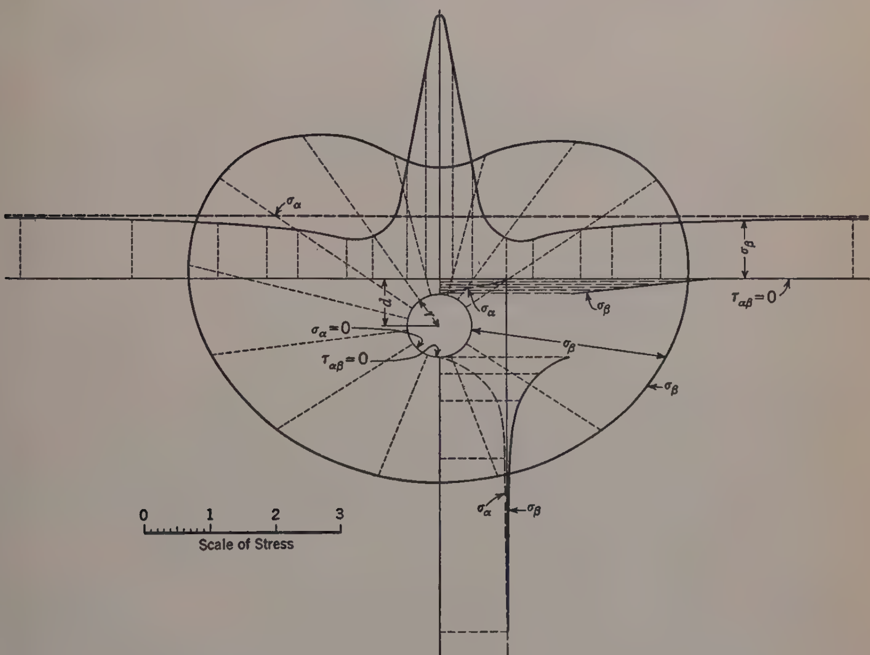


FIG. 9.—UNIT NORMAL COMPRESSION ON ALL EXTERNAL BOUNDARIES; $\frac{d}{r} = 1.5$

As stated previously, the computations made for a unit normal load acting on the outer boundary may also be used to determine the stresses in the founda-

[29] *Technical Memordum No. 597*, U. S. Bureau of Reclamation.

tion due to a unit normal load acting on the boundary of the hole. It is obvious that, if a unit normal load is applied simultaneously on both the outer and inner boundaries, all points in the body will have unit normal stress in all directions. Such a body is said to be in a hydrostatic state of stress. Principal stresses are equal to the same quantity at all points, and all shear stresses are zero.

Let a normal stress at any point due to a unit normal load acting on the boundary $\alpha = 0$ be denoted σ' and the shear by τ'. Furthermore, let a normal stress and shear at the same point due to a unit normal load acting on the boundary $\alpha = \alpha_2$ be denoted by σ'' and τ'', respectively. Then, if unit pressures act simultaneously on both boundaries, the hydrostatic state gives:

$$\sigma' + \sigma'' = +1 \qquad (77a)$$

and

$$\tau' + \tau'' = 0 \qquad (77b)$$

if compression stress is considered positive.

By solving Equations (77), the normal stress σ'' and the shears τ'' can be found in terms of σ' and τ', respectively, as follows:

$$\sigma'' = 1 - \sigma' \qquad (78a)$$

and

$$\tau'' = -\tau' \qquad (78b)$$

A negative result will then denote tension. Hence, if the normal stresses found for a unit normal load acting on $\alpha = 0$ are deducted from unity, the stresses due to a unit normal load acting on $\alpha = \alpha_2$ will be obtained. The shears found at any point of the body for a unit load acting on $\alpha = 0$ will be of equal magnitude but opposite direction for a unit load acting on $\alpha = \alpha_2$.

Equations (78) are easily solved graphically by curves such as those in Fig. 9. It will be found that the stresses σ_α'' are still compressive and attain their maximum value at the hole, diminishing asymptotically to zero away from the hole. The stresses σ_β'' become tension, maximum at the hole, and also approach zero asymptotically.

In order to obtain the stresses for a boundary loading of any magnitude, it is only necessary to multiply those obtained for the unit loading by the magnitude of the actual boundary load. In other words, the stress diagrams in Fig. 9 are influence lines in the usual engineering sense.

Finally, if the external boundaries are loaded with a normal compressive force P_0 and the perimeter of the hole with a normal compressive force, P_1, the solution is obtained by multiplying the stresses σ' and τ' by P_0 and the stresses σ'' and τ'' by P_1, and then superimposing the results.

It is to be noted that the stresses σ' and τ' are due to a unit compression on the entire external boundary $\alpha = 0$. This includes both the plane surface of the foundation and the infinite boundaries. In many practical applications it is desirable to remove all or part of the load on the infinite vertical boundaries. For this purpose the theoretical solution by Mr. Jeffery [11] (mentioned parenthetically by Mr. Mindlin in the text following Equation (36)), is applicable. However, the author has raised a doubt about the strict correctness of Mr.

Jeffery's application of his general bipolar solution to this case; therefore the writer has conducted a series of photoelastic experiments which will serve both a practical purpose and as an experimental check on Mr. Jeffery's theoretical results.

Photoelastic Study.—In the photoelastic study of Mr. Jeffery's problem, of course, it was impossible to use a bakelite sheet of infinite dimensions. It is believed that the results obtained by using a plate of finite dimensions give valid results when the diameter of the hole and the distance from the free edge are small in comparison with the over-all dimensions of the model, in accordance with Saint-Venant's principle.

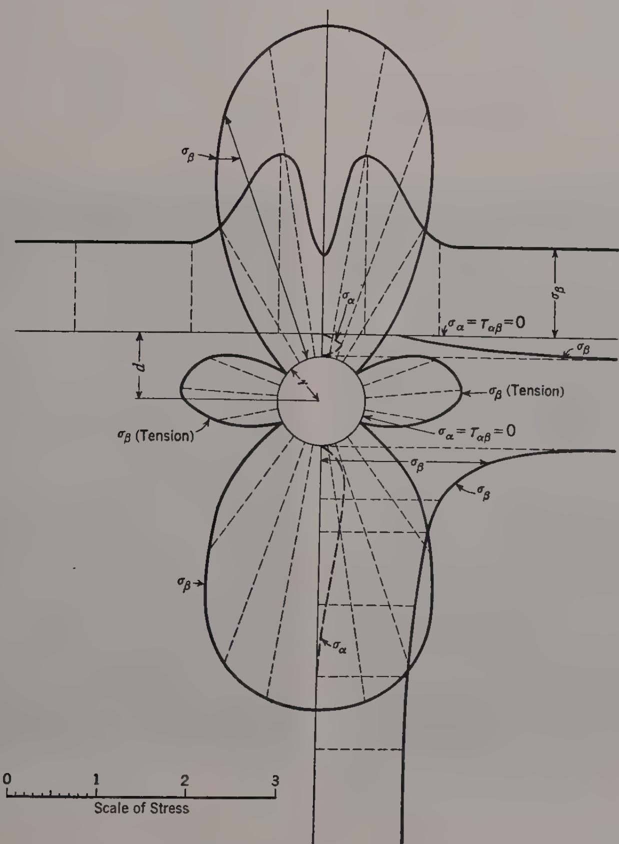
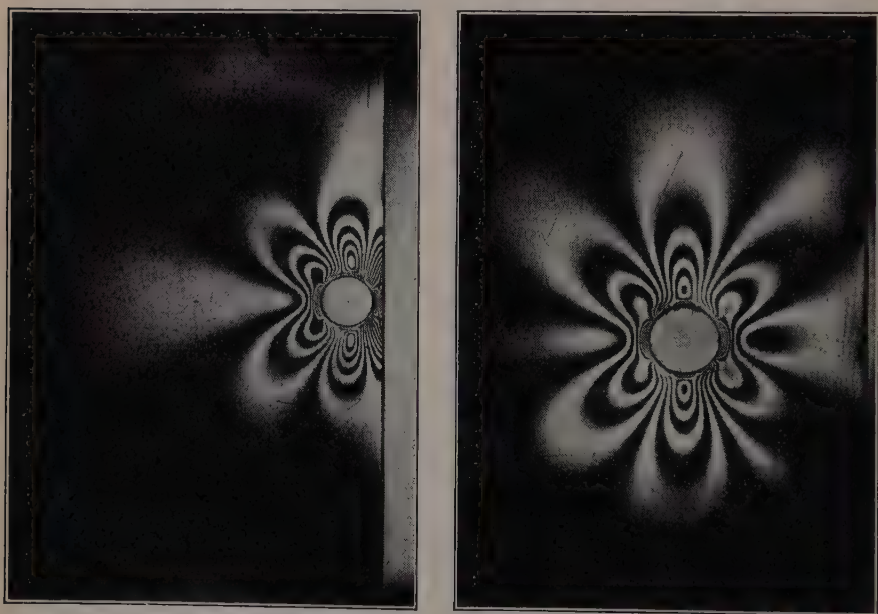


FIG. 10.—UNIT NORMAL COMPRESSION ON VERTICAL BOUNDARIES AT INFINITY; $\frac{d}{r}$ = 1.5 (ALL STRESSES COMPRESSION EXCEPT AS NOTED)

The model was constructed in rectangular form of bakelite, 0.5 in. thick, with vertical boundaries 2.75 in. wide by 4.62 in. long. A quarter-inch hole was drilled near one free edge of the model, such that $\frac{d}{r}$ was 6 (d is the distance from the center of the hole to the straight boundary and r is the radius of the

hole). When the experiment was completed for the case $\frac{d}{r} = 6.0$, the model was reduced in width successively so that the ratio $\frac{d}{r}$ was equal to 5.0, 4.0, 3.0, 2.0, 1.0 and 1.5. The normal load on the lateral boundaries was maintained at a constant intensity of 1 000 lb per sq in.

The photoelastic interferometer [30] was used to measure the individual principal stresses along the shortest straight line connecting the straight edge of the model with the hole. The boundary stresses around the hole and along the free edge were obtained directly from the isochromatic photographs.[29] For example, the stresses along these lines for $\frac{d}{r} = 1.5$ are shown in Fig. 10, and the isochromatic photographs for $\frac{d}{r} = 1.5$ and 6.0 are shown in Fig. 11. In these the calibration of principal stresses along a free boundary is 0.173 p per



(*a*) $\frac{d}{r} = 1.5$ (*b*) $\frac{d}{r} = 6.0$

FIG. 11.—ISOCHROMATIC PHOTOGRAPHS; MAXIMUM SHEAR CALIBRATION 0.0865 p PER FRINGE; VERTICAL LOAD

fringe, in which p is the intensity of loading on the vertical boundaries and is assumed equal to unity. The probable error in the experimental stresses is $\pm$ 5 per cent. The calibration for maximum shears, of course, would be 0.0865 p. Note that the stresses along the straight boundary are compression throughout.

[30] "Direct Optical Measurement of Individual Principal Stresses," by J. H. A. Brahtz and John Soehrens, Jun. Am. Soc. C. E., *Journal of Applied Physics*, Vol. 10, No. 4, April, 1939.

The stresses plotted as in Fig. 10 for unit end loading of the vertical infinite boundaries, in connection with the stresses plotted as in Fig. 9 and combined with the author's results, permit of the complete stress determination for various boundary loading and body forces.

The theoretical results of the paper may be used with sufficient accuracy for civil engineering purposes to estimate the stress conditions around vertical shafts and inspection galleries near the faces of the dam, and pressure pipes running through the dam. These problems have been treated elsewhere [31, 32] under the assumption that the openings were far removed from the faces, whereas the present theory permits the opening to come as close to the faces as practical considerations will permit. In this application of the theory, it is customary first to compute the principal stresses at the center of the opening, as if the opening did not exist, and then apply these principal stresses as normal boundary loading far removed from the opening and compute the stresses in the vicinity of the opening by the present theory.

It is hoped that the author, in his closure, will comment further on the apparent discrepancies in the theoretical and photoelastic results for a plate with an eccentric circular opening and under a uniform traction. This problem is important as it occurs in many forms in practical structures.

Finally, it should be noted that the author's bipolar transformation can be used also for the determination of the liquid pore pressures which may exist in the material surrounding the tunnel and the seepage into the tunnel. The theory and formulas for these effects may be found in the memorandum [29] as well as the details of the various theoretical and photoelastic studies mentioned and exemplified by the writer.

Acknowledgments.—The writer wishes to express his appreciation to the staff of the photoelastic laboratory and others of the Bureau of Reclamation, Denver, Colo., for their valuable assistance in the preparation of this discussion. Mr. John E. Soehrens, Associate Engineer, Mr. H. B. Phillips, Junior Engineer, and Mr. H. J. Kahm, Engineer Aid, performed the photoelastic experiments. Mr. C. N. Zangar, Assistant Engineer, and Mr. J. R. Bruggeman, were responsible for the mathematical analyses.

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[31] "Introduction to the Stresses in the Wall Rock of Tunnels," by Frederick C. Carstarphen, M. Am. Soc. C. E., *The Mines Magazine* (Colorado School of Mines, Golden, Colo.), May, 1939.

[32] "Stresses Around Circular Holes in Dams and Buttresses," by I. K. Silverman, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 133.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

COMBINING GEODETIC SURVEY METHODS WITH CADASTRAL SURVEYS

Discussion

By Gordon Macleish, Assoc. M. Am. Soc. C. E.

Gordon Macleish,[10] Assoc. M. Am. Soc. C. E. (by letter).[10a]—The engineer in charge of any extensive survey should find this paper of great interest. The methods described by the author for assuring accuracy in the work and for maintaining a complete historical record of all points established and co-ordinated are excellent; and the paper shows clearly the value of systematizing, carefully, the field and office procedures and arranging the observations and measurements so as to give the required information with the greatest economy of effort. It is to be hoped, also, that the paper will receive the attention it deserves from the legal profession and from the judiciary because, as the author well remarks, the indirect methods, although in many cases much more accurate than the orthodox "ground methods," are too often "beyond the previous knowledge of the Court."

The methods given by the author for the co-ordination of an intersection station and of a three-point fix (Figs. 6 and 7) are very concise and, if co-ordinates only are wanted, require a minimum of trigonometric functions to be looked up in tables and entered on the form. It often happens, however, that the bearings and distances are required as well as the co-ordinates; and in such cases it is necessary, of course, to look up the usual sine and cosine functions and to use additional lines for their computation; and still further work is required to check these quantities after they are computed.

It seems to the writer that the retention of tenths of seconds in angles and bearings in the computation of third-order intersections and three-point problems is a refinement that is scarcely warranted by the strength of such fixes. In Fig. 6 the location of the point from Inchelium and Dorr would be changed by only 0.01 ft, and in Fig. 7 the three-point fix would be changed by only 0.02 ft by using the nearest even seconds; and much time would be

Note.—This paper by Carl M. Berry, Assoc. M. Am. Soc. C. E., was published in September, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1939, by William L. Sawyer, Assoc. M. Am. Soc. C. E.; and January, 1940, by Philip Kissam, Assoc. M. Am. Soc. C. E.

[10] Cons. Engr., Santa Barbara, Calif.

[10a] Received by the Secretary January 8, 1940.

saved in the computations. When the use of tenths of seconds is justified it might be advisable to consider also the effect of the azimuth corrections in computing the longer fixes. A method of doing so will be given subsequently herein, whereby grid co-ordinates may be computed accurately from geodetic angles for lines of any length up to 25 miles or more, the length being limited only by the accuracy with which the azimuth correction terms can be computed.

The method given for the computation of an intersection station (Fig. 6) has the disadvantage of requiring the use of at least a ten-bank calculating machine unless the co-ordinates are all small; and even in that case the quantities m and b in Equation (1), or K in Fig. 7, may still be very large. This will be the case whenever the line in question is nearly north and south. It frequently happens, too, that an intersection or three-point fix is required in the course of a traverse, in which case it is very convenient to use a method of computation that utilizes the ordinary traverse form. The writer uses the following methods which he finds very quick and simple, and which he prefers for the reasons that:

1. A simple traverse form is used, whether for an intersection or a three-point, both of which are computed by the same process; and an eight-bank machine is large enough for all problems;
2. The calculations give, directly, the distances to, as well as the co-ordinates of, the unknown point, and are entirely self-checking for all quantities involved;
3. All trigonometric functions used are less than unity, thus requiring tables of tangents only up to 45°;
4. All digits on the left in either co-ordinate column that are common to all points may be disregarded without affecting any part of the work;
5. No unusual entries are required in the traverse form except the values of the tangents of the bearings; and
6. All three-point fixes are computed in the same manner whatever the relative positions of the points, the only effect of position being the addition of 180° to one or another of the bearings.

The time required by this method to calculate and check a three-point fix, including bearings and distances, is about the same as by the author's method; and for an intersection point it is probably a little less.

Consider first the solution for an intersection point, since this is the basis also of the three-point solution. Suppose Point P (Fig. 11) to be located by cuts from the known points A and B.

Let $\Delta x_{AB} = x_B - x_A$; $\Delta y_{AB} = y_B - y_A$; α and a = bearing and length of $A\,P$; and β and b = bearing and length of $P\,B$. It is evident from Fig. 11 that

$$\Delta x_{AB} - a \sin \alpha = (\Delta y_{AB} - a \cos \alpha) \tan \beta \qquad (5a)$$

and

$$\Delta x_{AB} - b \sin \beta = (\Delta y_{AB} - b \cos \beta) \tan \alpha \qquad (5b)$$

Whence,

$$a = \frac{\Delta x_{AB} - \Delta y_{AB} \tan \beta}{\sin \alpha - \cos \alpha \tan \beta} \qquad (6a)$$

or

$$a = \frac{\Delta y_{AB} - \Delta x_{AB} \cot \beta}{\cos \alpha - \sin \alpha \cot \beta} \quad \text{(6b)}$$

$$b = \frac{\Delta x_{AB} - \Delta y_{AB} \tan \alpha}{\sin \beta - \cos \beta \tan \alpha} \quad \text{(7a)}$$

or

$$b = \frac{\Delta y_{AB} - \Delta x_{AB} \cot \alpha}{\cos \beta - \sin \beta \cot \alpha} \quad \text{(7b)}$$

As shown subsequently, the method of using these formulas is such that only their general form need be remembered.

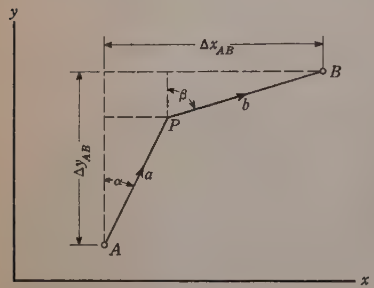


FIG. 11

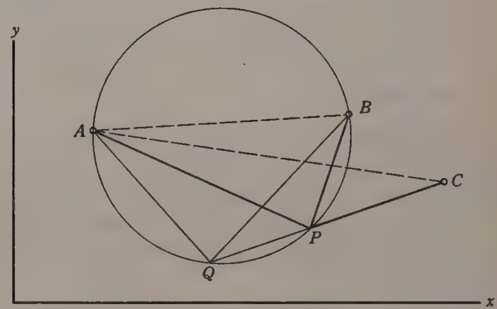


FIG. 12

The signs of the functions of the bearings are in strict accordance with the co-ordinate system used—that is, for the usual system as shown, a bearing in the northeast quadrant has all functions positive; a bearing in the southeast quadrant has the sine positive, the cosine negative, and hence the tangent $\left(\frac{\text{sine}}{\text{cosine}}\right)$ negative, etc. The distance a may be found from either of Equations (6); but in order to keep all quantities as small as possible Equation (6a) is used if β is less than 45° and Equation (6b) if β is greater than 45°. Thus, tables of tangents to seconds of arc beyond 45° are not needed, and no function is greater than unity. Knowing a, the co-ordinates of Point P can be calculated, whence b becomes known as that distance necessary on the bearing β to reach the co-ordinate of Point B.

By this method the solution of the author's intersection from Inchelium and Elburn in Fig. 6 (omitting the derivation of the bearings which can be inserted easily) would be made as shown in Table 2. The underlined digits in the co-ordinate columns need not be used in the computation.

Since β is less than 45°, use Equation (6a) and compute the denominator first, jotting it down. Then compute the numerator and without clearing it from the machine divide it by the denominator. This gives the value of a, thus,

$$a = \frac{(+\ 14\ 242.54) - (+\ 14\ 448.17)\ (-\ 0.1760271)}{(+\ 0.9633540) - (+\ 0.2682333)\ (-\ 0.1760271)} = 16\ 610.23.$$

Having found the value of a, enter the y-co-ordinate of Point A in the machine and compute the y-co-ordinate of P. Then without clearing the machine enter $\cos \beta$ on the keyboard and crank up enough turns (positively or negatively) to arrive at the y-co-ordinate of Point B, the value of the distance b being given by the number of turns cranked. The x-co-ordinate column then serves as a check on the work and gives the x-co-ordinate of P in doing so. The distance b may be calculated by means of either co-ordinate column, the other serving as a check; but for greatest accuracy that column should be used which corresponds to the larger of the two functions (cosine or sine) of β. The only quantities not checked by the foregoing method are the entries of $\cos \alpha$ and $\sin \alpha$. Hence these should be verified or, as a final check, the value of b should be computed from Equation (7a) or (7b). If b is computed originally from Equation (7a) or (7b), the two co-ordinate columns frnish au double check on the entire work.

TABLE 2.—COMPUTATION OF INTERSECTION STATION

Point	Bearing	Distance, in feet	cos sin (tan, cot)	Y	X
A. Inchelium				479 159.04	2 639 946.87
	N. 74° 26′ 27.″0 E.	16 610.23	0.2682333 0.9633540 (+0.2784369)	+	+
P. X-station				483 614.46	2 655 948.40
	N. 9° 59′ 00.″0 W.	10 146.39 ✓	0.9848582 0.1733617 (−0.1760271)	+	−
B. Elburn				493 607.21	2 654 189.41 ✓
				$\Delta y_{AB} = +14\,448.17$	$\Delta x_{AB} = +14\,242.54$

If the value of the denominator in Equation (6) or (7) is negative the numerator will be negative also, provided that the bearings are consistent with the given co-ordinates. In this case leave the numerator as a negative entry in the machine, enter the absolute value of the denominator on the keyboard and crank up positively until the result is zero. The value of a (or b) will then be given by the number of turns cranked. Note that the first term of the denominator in either of Equations (6) is the same function (cosine or sine) of α as the smaller of the two functions (cosine or sine) of β, and that the first term of the numerator is from the corresponding co-ordinate column; and this applies similarly for the use of Equations (7). If these two reminders are fixed firmly in mind Equations (6) and (7) need be remembered only as to general form, and the proper one will be chosen and used correctly almost without thought.

Much time can be saved in the computation by carrying the co-ordinates (or their last seven or eight digits) continuously in the calculating machine and letting the multiplications accumulate, either positively or negatively as required. This saves writing down the individual values of Δx and Δy unless they are needed to compute a length or a bearing. They are never needed to compute areas, since cross-multiplication of co-ordinates (dropping the common digits on the left) gives the area much more quickly and with no additional

columns of figures required. Thus the area of the triangle Inchelium-P-Elburn is:

$$\tfrac{1}{2}\,[(39\,946.87 \times 83\,614.46) + (55\,948.40 \times 93\,607.21) + (54\,189.41 \times 79\,159.04) - (79\,159.04 \times 55\,948.40) - (83\,614.46 \times 54\,189.41) - (93\,607.21 \times 39\,946.87)] = 83\,868\,164 \text{ sq ft.}$$

This computation may be made without clearing or reading the machine until the double-area is reached. The sign of the result may be disregarded.

The following solution of the three-point problem is an analytical adaptation of a well-known graphical method, and consists almost entirely of two simple intersection computations exactly as in the foregoing.

Let Point P (Fig. 12) be located by measuring the angles subtended at P by the three known points A, B, and C. These angles may be given directly or as a list of directions from any initial. Choose, as A and B, those two of the three known points which subtend at P the angle nearest to 90°, and label them so that $A \longrightarrow B$ is clockwise about P. Call the third known point C. The auxiliary point Q is the other intersection of the line $P\,C$ with the circle through A, B, and P. Denoting azimuths by α and directions by D, with subscripts for identification:

$$\alpha_{AQ} = \alpha_{AB} + (D_{PC} - D_{PB}) \quad \ldots\ldots (8a)$$

($\pm$ 180° if P is within triangle $A\,B\,C$, or if C is counterclockwise < 180° from A);

$$\alpha_{QB} = \alpha_{AB} + (D_{PC} - D_{PA}) \quad \ldots\ldots (8b)$$

($\pm$ 180° if P is within triangle $A\,B\,C$, or if C is clockwise < 180° from B). The azimuths in terms of the angles will be evident from Equations (8). Knowing these two azimuths Point Q may be co-ordinated as an intersection point by the foregoing method using Equations (6). Compute α_{QC} from differences of co-ordinates. Then,

$$\alpha_{CP} = \alpha_{QC} \pm 180° \quad \ldots\ldots (9)$$

(or $= \alpha_{QC}$ if C is between P and Q);

$$\alpha_{PA} = \alpha_{CP} \pm 180° - (D_{PC} - D_{PA}) \quad \ldots\ldots (10a)$$

and

$$\alpha_{PB} = \alpha_{CP} \pm 180° - (D_{PC} - D_{PB}) \quad \ldots\ldots (10b)$$

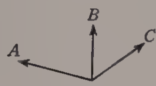
Use Equation (10a) or (10b) whichever gives the best intersection at P with the line $C\,P$—that is, the angle at P nearest to 90°. Azimuths and angles may be carried forward easily on the calculating machine, adjusting degrees, minutes, and seconds for carry-over only at the end of the operation. By doing this repeated entries are avoided.

Point P is now co-ordinated as an intersection point as described. A final check is made by computing the third course from P by differences of co-ordinates and comparing the computed angle with the observed value. The agreement should be exact.

It is usually not known without some thought whether or not C is between P and Q, but this is of small consequence. The value of the length $C\,P$ as computed from Equation (6) should be positive; and if it comes out negative its absolute value is still correct, but the recorded directions of $C\,P$ and $P\,A$ (or of $C\,P$ and $P\,B$ if closure is made on Point B), together with the signs of their functions, should be reversed. These will be the only changes necessary, since the error in the assumed position of C will be apparent before any other value is computed.

Applying this method to the author's three-point fix given in Fig. 7, the solution would be as shown in Table 3. The underlined digits in the co-ordinate columns can be ignored in the computation, any changes in them being recorded by the machine.

TABLE 3.—COMPUTATION OF THREE-POINT FIX



	Directions	Angles
A. Six Mile	0° 00′ 00″.0	
		75° 43′ 02″.5
B. Nine Mile	75° 43′ 02″.5	
		52° 23′ 25″.4
C. Jensen	128° 06′ 27″.9	

Point	Azimuth	Length	cos sin (tan, cot)	Y	X
A. Six Mile				371 020.42	2 598 804.10
	215° 40′ 05″.8	5 498.24	0.8124065 0.5830915 (+0.7177337)	+	+
Q.				375 487.23	2 602 010.08
±180°	111° 23′ 08″.3	5 535.52✓	0.3646434 0.9311472 (−0.3916066)	+	−
B. Nine Mile				377 505.72✓	2 596 855.69
A–B	163° 16′ 40″.4		Δy_{AB}	= +6 485.30	Δx_{AB} = −1 948.41
Q–C	150° 00′ 49″.3		Δy_{QC}	= +12 181.50	Δx_{QC} = −7 029.11✓
C. Jensen				387 668.73	2 594 980.97
				−	+
±180°	330° 00′ 49″.3	12 401.72	0.8661449 0.4997930 (−0.5770316)		
P. Point Near				376 927.04	2 601 179.26
	97° 37′ 23″.9	4 362.13✓	0.1326596 0.9911617 (−0.1338425)	+	−
B. Nine Mile				377 505.72✓	2 596 855.69
C–B			Δy_{CB}	= −10 163.01	Δx_{CB} = +1 874.72
P–A	21° 54′ 21″.5	6 366.28	0.9277974 0.3730845 (+0.4021183)	−5 906.62	−2 375.16✓
P–B	97° 37′ 23″.9				
Angle: APB					
Computed	75° 43′ 02″.4✓				
Measured	02″.5				

The clockwise angle, Six Mile-P-Nine Mile (= 75° 43′ 02″.5), is the nearest to 90°; so Six Mile is taken as Point A and Nine Mile as Point B, making Jensen Point C. It is evident immediately that P lies outside the triangle $A\,B\,C$, and that C is clockwise < 180° from B. The azimuth of $A\,B$ is computed from Δx_{AB} and Δy_{AB}. The azimuths of $A\,Q$ and $Q\,B$ are found from Equations (8) by simple addition, using the machine to avoid duplication

of entries in the form; the azimuth of $Q\,C$ is computed by co-ordinate differences; and the azimuths of $C\,P$ and $P\,B$ are found by addition using Equations (9) and (10*b*). The remainder of the work is identical with the foregoing for an intersection computation, both the points Q and P being co-ordinated in this manner using Equations (6) and (7).

If a fourth point D is observed as a check, compute the course $P\,D$ by co-ordinate differences and compare the calculated angle from one of the other points with its observed value. It is assumed that P will be located by the three points that give the strongest fix.

For a given set of angles the strength of a fix is dependent on the relative length of the line $Q\,C$ in comparison with $P\,A$ and $P\,B$, and decreases with a decrease in the relative length of $Q\,C$, the problem being indeterminate when $Q\,C$, or $P\,C$, is zero. When P is on the circle through A, B, and C either Point Q or Point P is coincident with C. If Q and C are nearly coincident the fix is weak. On the other hand, if P is nearly coincident with C the fix will be strong if Q is sufficiently distant from C to give the required precision to the computed azimuth of $Q\,C$ (the strength increasing also, of course, as the measured angles approach 90°); for in this case the computed position of P is comparatively insensitive to changes in the observed direction of $P\,C$. A three-point fix with reasonable angles observed from a point close to C (the directions of $P\,A$ and $P\,B$ being accurately measured and the direction $P\,C$ being measured approximately) is a strong fix. Hence, the usual statement that P should never be close to the circle through A, B, and C needs qualification.

Effect of Azimuth Correction Terms.—Because the azimuth correction terms in either the Lambert or the transverse Mercator projection are different for each direction observed, a geodetic or measured angle is not exactly represented by the difference between the corresponding grid azimuths. These correction terms are small and in most cases can be neglected entirely; but with long lines and high precision work in certain districts they may be large enough to be worth considering.

Since the approximate grid co-ordinates of the points must be known before the azimuth correction terms can be computed, two methods of procedure to include their effects are available: (*a*) A preliminary graphical solution may be made on a good map and the corrections computed from scaled quantities (the measured angles are then corrected and a final analytical solution made directly); and (*b*) a preliminary analytical solution is made first, using the observed angles. In Method (*b*) the azimuth corrections are computed from the resulting approximate grid co-ordinates; and from these are computed corrections to the preliminary co-ordinates of the unknown point which will give its final co-ordinates.

The azimuth corrections in seconds of arc are given by the formulas:[11]

$$\epsilon\ (\text{Lambert}) = -\frac{x_2 - x_1}{2\,\rho_0^2 \sin 1''}\left(y_1 - y_0 + \frac{y_2 - y_1}{3}\right) \quad \ldots\ldots(11a)$$

and

$$\epsilon\ (\text{transverse Mercator}) = +\frac{(y_2 - y_1)\,(2\,x_1' + x_2')}{(6\,\rho_0^2 \sin 1'')_g} \quad \ldots\ldots(11b)$$

[11] *Special Publication No. 193* (Manual of Plane-Coordinate Computation), U. S. Coast and Geodetic Survey, pp. 13 and 171.

in which the sub-one subscripts refer to the station at which the azimuth originates and the sub-two subscripts refer to the station at which the azimuth terminates. These corrections are to be applied to the grid azimuths with the signs as given. For an intersection computation the azimuths, α, originate at the fixed points where the cuts are made, whereas for a three-point computation they originate at the unknown point. Corrections must be computed for both sides of each angle used—that is, a correction for each direction.

Correction of an Intersection Station.—Using literal subscripts to denote the origin and terminus, the letter R denoting any reference point (for example, ϵ_{AR} is the correction to the grid azimuth from A to R), the formula for an intersection is:

$$\alpha_{AP} \text{ (grid)} = \alpha_{AR} \text{ (grid)} + \text{Angle } R\,A\,P \text{ (observed)} + (\epsilon_{AR} - \epsilon_{AP}) . \text{(12}a\text{)}$$

and

$$\text{Angle } R\,A\,P \text{ (grid)} = \text{Angle } R\,A\,P \text{ (observed)} + (\epsilon_{AR} - \epsilon_{AP}) \ldots \text{(12}b\text{)}$$

Then the change in the azimuth of $A\,P$ (which equals the change in the azimuth of $P\,A$) due to the azimuth corrections at A is

$$\Delta\alpha_{AP} = \Delta\alpha_{PA} = (\epsilon_{AR} - \epsilon_{AP}) \ldots\ldots\ldots\ldots\ldots \text{(13}a\text{)}$$

and, similarly,

$$\Delta\alpha_{BP} = \Delta\alpha_{PB} = (\epsilon_{BR} - \epsilon_{BP}) \ldots\ldots\ldots\ldots\ldots \text{(13}b\text{)}$$

Furthermore

$$\tan^{-1} \frac{x_A - x_P}{y_A - y_P} = \alpha_{PA} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14)}$$

and, taking the total differential with x_P and y_P variable, and substituting $x_A - x_P = s_{PA} \sin \alpha_{PA}$; $y_A - y_P = s_{PA} \cos \alpha_{PA}$; and $(x_A - x_P)^2 + (y_A - y_P)^2 = s_{PA}{}^2$:

$$\frac{\cos \alpha_{PA}}{s_{PA}}\, dx_P - \frac{\sin \alpha_{PA}}{s_{PA}}\, dy_P = - d\alpha_{PA} \ldots\ldots\ldots\ldots \text{(15)}$$

Changing to increments and dividing by sin 1″ to change the value of $\Delta\alpha$ from radians to seconds of arc:

$$\frac{\cos \alpha_{PA}}{s_{PA} \sin 1''}\, \Delta x_P - \frac{\sin \alpha_{PA}}{s_{PA} \sin 1''}\, \Delta y_P = - \Delta\alpha_{PA}{}'' = - \epsilon_{AR} + \epsilon_{AP} \ldots \text{(16}a\text{)}$$

Similarly,

$$\frac{\cos \alpha_{PB}}{s_{PB} \sin 1''}\, \Delta x_P - \frac{\sin \alpha_{PB}}{s_{PB} \sin 1''}\, \Delta y_P = - \Delta\alpha_{PB''} = - \epsilon_{BR} + \epsilon_{BP} \ldots \text{(16}b\text{)}$$

As always, the signs of the functions are in accordance with the co-ordinate system. Equations (16) may be solved simultaneously for Δx_P and Δy_P, the coefficients being based on the results of the preliminary solution and the values of the ϵ's computed from Equation (11a) or (11b) according to the projection in use. A slide-rule may be used for the entire work.

Correction of a Three-Point Station.—At a three-point station the azimuth corrections are all based on the unknown Point P as origin; so if the directions are measured from an extraneous initial no correction need be computed for

the initial since it will cancel out of the equations. Corrections will be required to the azimuths from P to each of the three fixed points A, B, and C, and to any additional point that is used for a check. The changes in the angles P_{AB} and P_{BC} due to the corrections are, in seconds of arc, $\Delta P_{AB} = \epsilon_{PA} - \epsilon_{PB}$; and $\Delta P_{BC} = \epsilon_{PB} - \epsilon_{PC}$. Angle $P_{AB} = \alpha_{PB} - \alpha_{PA}$, and applying Equation (14):

$$\tan^{-1}\frac{x_B - x_P}{y_B - y_P} - \tan^{-1}\frac{x_A - x_P}{y_A - y_P} = P_{AB} \quad \text{.............(16)}$$

whence, by a process similar to that used in deriving Equations (16):

$$\left(\frac{\cos\alpha_{PA}}{s_{PA}\sin 1''} - \frac{\cos\alpha_{PB}}{s_{PB}\sin 1''}\right)\Delta x_P - \left(\frac{\sin\alpha_{PA}}{s_{PA}\sin 1''} - \frac{\sin\alpha_{PB}}{s_{PB}\sin 1''}\right)\Delta y_P$$
$$= -\Delta P_{AB} = -\epsilon_{PA} + \epsilon_{PB} \quad \text{................(17a)}$$

Similarly,

$$\left(\frac{\cos\alpha_{PB}}{s_{PB}\sin 1''} - \frac{\cos\alpha_{PC}}{s_{PC}\sin 1''}\right)\Delta x_P - \left(\frac{\sin\alpha_{PB}}{s_{PB}\sin 1''} - \frac{\sin\alpha_{PC}}{s_{PC}\sin 1''}\right)\Delta y_P$$
$$= -\Delta P_{BC} = -\epsilon_{PB} + \epsilon_{PC} \quad \text{................(17b)}$$

The coefficients are computed from the preliminary solution and the ϵ-values from Equation (11). Equations (17), solved simultaneously for Δx and Δy, give the corrections to the preliminary co-ordinates of P due to the azimuth corrections, a slide-rule being used throughout.

The writer is glad to observe a more extensive use of intersections and three-point fixes in geodetic and cadastral surveys that are based on accurate triangulation, provided that all such locations are checked independently as was required in the surveys described by the author. Used with discrimination and with a facile knowledge of the relations between form and strength of figure, they constitute a valuable means of locating the points whose positions are required, quickly, and with considerable, if not with the highest, precision. They are thus particularly suited to cadastral, topographic, photogrammetric, and non-urban property surveys.

The indirect methods of surveying, along with triangulation in general and conformal projection in particular, will acquire the legal and judicial sanction accorded to direct measurements and plane representation as they become more generally known, both as to detail processes and as to the results. Mr. Berry's paper is a valuable contribution toward the increase of this knowledge.

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DISCUSSIONS

AN IMPROVED METHOD FOR ADJUSTING LEVEL AND TRAVERSE SURVEYS

Discussion

By Messrs. R. M. Wilson, Cleveland B. Coe, and George D. Whitmore and C. C. Miner and W. O. Byrd

R. M. Wilson,[10] M. Am. Soc. C. E. (by letter).[10a]—In 1927 a readjustment of the first-order triangulation net covering the United States was begun by the U. S. Coast and Geodetic Survey. The result changed, slightly, the geodetic coordinates for established triangulation stations throughout the country. Corrections to obtain proper agreement with the re-defined geodetic reference datum were thus imposed upon all dependent surveys which had used those stations before the readjustment.

The U. S. Geological Survey had in its files the accumulated results of its own control surveys, consisting of triangulation done since 1879 and transit traverse done since 1899, all referred to the old North American Datum. Because of the great volume of records involved, it was a real problem to find an economical way to revise the results of these surveys to make them consistent with the new North American Datum of 1927. The scheme adopted to refer the older triangulation to the new datum has been described by Mr. Speert in another paper.[11]

The older transit traverse by the Geological Survey had been extended into areas far from supporting triangulation. Now, however, arcs of new triangulation by the Coast and Geodetic Survey pass through many areas where support for the older traverse was needed urgently. In 1936, an allotment of funds by the U. S. Public Works Administration made it possible to send out field parties to make ties between the older traverse and the new triangulation. Thus the results of the traverse required revision not only because of changes in

Note.—This paper by Clarence Norris, Esq., and Julius L. Speert, Assoc. M. Am. Soc. C. E., was published in October, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1940, by Messrs. George H. Dell, and Howard S. Rappleye.

[10] Chf., Section of Computing, U. S. Geological Survey, Washington, D. C.

[10a] Received by the Secretary December 22, 1939.

[11] "Readjustment of Triangulation Datum," by Julius L. Speert, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1002.

the coordinates of triangulation stations from which the traverse was originally extended, but also because of the strengthened ties to new arcs of triangulation.

Generally the traverse in a local project was given simple line-by-line adjustment as soon as it was computed, and much of it was subsequently readjusted one or more times due to the influence of newer traverse in later adjacent projects. In order that the effect of these old and sometimes doubtful local adjustments may be eliminated in the general revision to the new datum, the separate lines or elements of the traverse net are now (1940) being taken from the records as originally computed, freed from the corrections of all previous adjustments. Then they can be fitted together again, simultaneously and upon equal terms, under the present conditions of stronger basic control.

A suitable and conventional method of least squares was already available when these general revision adjustments were begun about 1934. Elements of the net were assembled into groups until a convenient number of circuits were formed and condition equations, one for each circuit, were solved in the usual manner. The volume of work thus accomplished is indicated by the following data. The number of circuits in one solution usually was about fifteen, although a few contained as many as twenty-five, and one contained thirty-six. About 180 such solutions were completed, all together involving some 40,000 linear miles of transit traverse surveys.

Out of that background the Norris-Speert method emerged. Now it is the number of unfixed junction points involved, rather than the number of circuits, that determines the size of an individual solution. The elements to be adjusted are assembled into groups as before, but with about twenty equations preferred instead of fifteen; in two instances, groups containing more than fifty equations have been solved. Since 1938 about seventy-five of these solutions have been done, so that nearly 18,000 additional miles of traverse lines have been adjusted by the new method. Approximately 43% of the traverse done before the results of the 1927 triangulation adjustment were available still needs readjustment, however. The Norris-Speert method has been used also to adjust a large part of the new traverse work of the Geological Survey, at the rate of about 7,000 miles per yr. It will be seen, therefore, that the method has been fully tested in practice.

The paper seems well written, and it is difficult to imagine how the ideas it contains could be more clearly expressed. Nevertheless, to one familiar with the computation there comes a feeling of regret that a really simple routine should be so masked, by the necessity of conveying it in print, as possibly to hide its simplicity from any one not having the perseverance to feel his way through it the first time.

The basic principle, which can be grasped readily by any one familiar with elementary algebra, is stated in the third paragraph of Appendix I thus: "* * * the adjusted elevation of bench mark D is to be a weighted mean of the elevations brought in through the various lines tying to it." This is easy to achieve if the "lines tying to it" all come from points whose elevations are fixed and known. The condition may be expressed as a simple algebraic equation. If a line comes from another point (called G, perhaps), where the elevation is not known, the equation can still be written for the elevation of D

by including the elevation of G as an unknown. Another equation can then show the elevation of G with the elevation of D as an unknown. Then these two simple algebraic equations can be solved simultaneously for the unknown elevations at the two points D and G. Extending this principle, simultaneous equations, properly weighted, can be written for all unadjusted junction points of a complicated net. Solving them, the elevation determined at each junction will then be "* * * a weighted mean of the elevations brought in through the various lines tying to it." That principle is the important disclosure; the remainder of the paper is a description of the computing forms and routines as used by the Geological Survey to apply the principle efficiently in practice.

Cleveland B. Coe,[12] Assoc. M. Am. Soc. C. E. (by letter).[12a]—In presenting a method that may be used by any surveyor, to adjust the results of his work, the authors of this paper are to be commended greatly. Unfortunately, it is true that surveyors and designers alike tend to "shy away" from anything that suggests higher mathematics, and the name "least squares" is something to terrify the unwary. Consequently, the presentation of a simplified method is a genuine service to the profession. The writer searched many texts without success for a simple method of adjustment for a series of interlocking traverses in connection with a fairly extensive contour survey of the town of Norris, Tenn., and the Norris-Speert presentation would have been extremely welcome.

Inasmuch as the application given in the paper is to traverse closures, the writer feels that the paper would be made more valuable to the average engineer by an example of adjustment in a very simple and typical case of levels run to establish bench marks along a surveyed route. For this purpose consider the notes in Table 8, reduced from a series of level lines. This is the common case of running from one fixed bench mark, closing on another, and establishing between them a number of new marks; then running back, turning

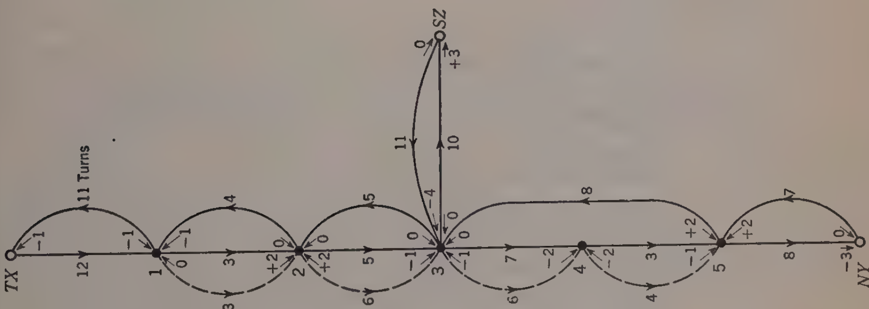

Fig. 6

on each mark, and closing on the original. A third set of readings is provided when the cross sections are taken and an additional tie to a third fixed bench mark also enters the adjustment. For clearness, the field procedure and closure results are shown in Fig. 6. For convenience, the closure results,

[12] Associate Highway Engr. (TVA), Chattanooga, Tenn.

[12a] Received by the Secretary January 8, 1940.

originally in hundredths of a foot, are shown as integers. The elevations obtained by the first line run (shown in Fig. 6 as straight) are taken as the basic elevations, to which any computed correction is to be applied. Therefore, all closures on this line are zero except the final closures on fixed bench marks.

The lines are weighted inversely as the number of turns. This is a reasonable assumption, as the foresights and backsights are presumed to be equal and not longer than 300 ft. It is assumed further that the cross-section levels are not taken as carefully as the bench levels and therefore the weights are made, arbitrarily, one half of those of the bench and check levels, as given in Table 9. These weights are obtained by setting 3 on the CI-scale of the slide rule over 1 on the D-scale and reading directly under each of the other number of turns.

The table is then filled out in accordance with the Norris-Speert method, following these general rules (see Table 4);[12b]

TABLE 8.—LEVELING NOTES

Bench Mark	Bench Levels: Number of turns	Bench Levels: Elevation
TX*(El. 72.22)		72.22
	12	
1		85.06
	3	
2		80.44
	5	
3		95.83
	7	
4		65.37
	3	
5		42.08
	8	
NY*(El. 25.01)		24.98
Check Levels		
NY		25.01
	7	
5		42.10
	8	
3		95.83
	5	
2		80.44
	4	
1		85.05
	11	
TX		72.21
Tie to Third Fixed Bench Mark		
3		95.83
	10	
SZ*(El. 76.16)		76.19
Check Levels		
SZ		76.16
	11	
3		95.79
Cross-Section Levels		
1		85.06
	3	
2		80.46
	6	
3		95.82
	6	
4		65.35
	4	
5		42.07

* Fixed bench mark.

1. Fill in all a-lines:
   - 1a-1 Negative sum of all weights entering 1,
   - 1a-2 Sum of weights entering 2 from 1; and similarly for 2a-2, 2a-3, etc.
2. Fill in N-column:
   - 1a-N Sum (weights × net closures) entering 1; and similarly for 2a-N, etc.
   - Check: Sum (weights × net closures) entering fixed bench marks. Place this sum at the bottom of N-column and add all items in the column, which gives zero.
3. Fill in S-column:
   - 1a-S Negative sum of weights entering 1 from a fixed bench mark, plus 1a-N; and similarly for 2a-S, etc.
   - Check: 1a-S = sum of all entries in line 1a;
     2a-S = sum of all entries in line 2a, plus all in Column (2); and similarly for 3a-S, etc.

[12b] Correction for *Transactions:* In Tables 3 and 4, reverse the signs of the three items at the bottom of Columns N and E.

TABLE 9.—WEIGHTS APPLIED TO LEVEL NOTES

Levels	WEIGHTS, FOR THE FOLLOWING NUMBER OF TURNS:								
	3	4	5	6	7	8	10	11	12
Bench and check levels.....	1.00	0.75	0.60	0.50	0.43	0.38	0.30	0.27	0.25
Cross-section levels.........	0.50	0.38	....	0.25	....	....	....	....	....

TABLE 10.—ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	N	S
(1) *ab*		−2.77	+2.25				−1.75	−2.27
c								
(2) *a*			−3.70	+1.45			+2.50	+2.50
b								
c								
(3) *a*				−3.08	+0.68	+0.38	−3.24	−3.81
b								
c								
(4) *a*					−2.06	+1.38	−0.63	−0.63
b								
c								
(5) *a*						−2.57	+3.14	+2.33
b								
c								
							−0.02	

This completes the original entries in the table and the results are shown in Table 10.

The problem is then solved by the Doolittle method in accordance with the following general rules (see Table 11):

1. 1*b*-R = negative reciprocal of 1*b*-1;
2. Multiply all entries in line 1*a* by 1*b*-R and enter results in line 1*c*;
3. 2*b*-2 = 1*b*-2 × 1*c*-2 + 2*a*-2;
4. If there are any other entries in line 1*b*, multiply them by 1*c*-2 and add to the next *a*-entry below to get the next *b*-entry;
5. 2*b*-R = negative reciprocal of 2*b*-2;
6. Any final *b*-entry is the sum of the products of all *b*-entries and *c*-entries above it, plus the last *a*-entry;
7. Repeat until table is complete;
8. All *b*-lines in column N are filled by multiplying all *b*-entries above by the *c*-entries in the column in question and adding the *a*-entry;
9. Similarly for the *b*-lines in column S;
10. Compute the *c*-entries in columns N and S—1*c*-N = 1*b*-N × 1*b*-R and 1*c*-S = 1*b*-S × 1*b*-R—and similarly for 2*c*-N, etc.; and
11. Check the *b*-lines and *c*-lines by adding across. The S-column is the sum of the other columns.

Solve backwards for each individual correction, shown at bottom of Table 11, as follows:

1. Correction at last bench mark is the same as the last *c*-entry in the N-column;
2. To obtain any correction, multiply all previously determined corrections by the *c*-entries in the line of the wanted correction and add to the N-value in that line.

Check: Fill out the CS-line by the same process, using the S-entries.

3. Then CN − CS = + 1.

TABLE 11.—SOLUTION OF ADJUSTMENT EQUATIONS

Equation	R	(1)	(2)	(3)	(4)	(5)	N	S
(1) *ab*	+0.361	−2.77	+2.25				−1.75	−2.27
c		−1.0	+0.812				−0.632	−0.820
(2) *a*			−3.70	+1.45			+2.50	+2.50
b	+0.535		−1.87	+1.45			+1.08	+0.66
c			−1.0	+0.775			+0.576	+0.353
(3) *a*				−3.08	+0.68	+0.38	−3.24	−3.81
b				−1.96	+0.68	+0.38	−2.40	−3.30
c				−1.0	+0.347	+0.194	−1.223	−1.682
(4) *a*					−2.06	+1.38	−0.63	−0.63
b	+0.549				−1.82	+1.51	−1.46	−1.77
c					−1.0	+0.830	−0.801	−0.971
(5) *a*						−2.57	+3.14	+2.33
b	+0.800					−1.25	+1.46	+0.21
c						−1.0	+1.168	+0.168
							−0.02	
CN (in 0.1 ft)		−0.753	−0.150	−0.938	+0.169	+1.168		
CS		−1.752	−1.149	−1.938	−0.831	+0.168		
CN in feet		−0.01	−0.00	−0.01	+0.00	+0.01		
Basic elevation		85.06	80.44	95.83	65.37	42.08		
Final elevation		85.05	80.44	95.82	65.37	42.09		

Attention is called to the results immediately adjacent to the fixed bench marks; namely:

From	To	Difference, in feet by survey
TX	1	12.84
1	TX	12.84
SZ	3	19.63
3	SZ	19.64
NY	5	17.09
5	NY	17.10

A common inference is that the correct differentials between the foregoing bench marks are 12.84, 19.635, and 17.095, respectively, and many surveyors

would start their adjustment of the net on this assumption. The computation of all corrections simultaneously shows the fallacy in this reasoning, as the most probably correct differentials prove to be 12.83, 19.66, and 17.08.

Inasmuch as the final elevations in this case are only to the nearest 0.01 ft, slide-rule computations were used throughout.

The writer agrees that the slight additional time required to adjust a circuit is well worth while and hopes that the average surveyor will be helped to grasp the mechanical process by the additional specific example set forth in this discussion. Acknowledgment is made to Mr. Speert for his suggestions as to the application of the new method to this typical example.

George D. Whitmore,[13] M. Am. Soc. C. E., C. C. Miner,[14] Assoc. M. Am. Soc. C. E., and W. O. Byrd,[15] Esq. (by letter).[15a]—The method of traverse and level adjustment described by the authors is correctly termed an "improved" and "simplified" method. Since April, 1939, these methods have been used consistently by the computing personnel of the Tennessee Valley Authority (TVA), to the exclusion of all other methods, for all traverse and level surveys of third-order or less accuracy. The authors infer, and the writers agree, that the described method may not always be ideally suited for adjustment of "most precise geodetic traverse" (first-order accuracy), since in traverse control of such precision, it is not always feasible to assume that all closure errors remaining after an azimuth adjustment are accidental errors of latitude and departure. In the cases of first-order and second-order level networks, the TVA computers continue to use the least-squares procedure recommended by the U. S. Coast and Geodetic Survey.

A point which always should be included in a paper on surveys adjustments is this: If the results of a traverse survey are to be published for use by other organizations and individuals, it is suggested that an inverse computation should always be made for each course of the traverse, from the finally adjusted coordinates of each pair of adjacent traverse stations, to determine an azimuth and a distance for each course which will agree exactly with the published adjusted coordinates. Otherwise, the preliminary or unadjusted azimuth and distance usually will disagree slightly with the values computed from adjusted coordinates, often to the confusion of the casual user of the traverse survey.

The writers concur with the authors' statements that: (1) Results by the "improved" method are identical with results by the conventional least-squares method; (2) results by the "improved" method usually seem more logical and more consistent than results by the junction-point method; and (3) the labor involved in using the "improved" method is not greater and is often less than the labor required for any cruder method.

The writers are glad to note that the authors have stressed the use of a diagram as a desirable preliminary of this method. A detailed diagram showing the magnitude and signs of closures, direction of closures, length of line, or other data fixing weights, is almost an absolute prerequisite in any method of adjust-

[13] Chf. of Surveys, Maps and Surveys Div., TVA, Chattanooga, Tenn.

[14] Chf. Survey Computer, TVA, Chattanooga, Tenn.

[15] Asst. in Chg. of Computing, TVA, Chattanooga, Tenn.

[15a] Received by the Secretary January 15, 1940.

ment. As emphasized by the authors, such a diagram shows at once the lines having largest closures, and the locations of lines most likely to contain the large errors. Most really experienced survey computers appreciate this and make use of adjustment diagrams accordingly. There is a decided tendency among inexperienced computers, however, to try to work without a diagram on which all essential data are shown. Hence they often become hopelessly confused and end with results far different from those which the mere inspection of a diagram would reveal to be the most logical. If it does nothing else, the diagram gives the computer an over-all or "bird's-eye view" of the system which he is adjusting—certainly a desirable thing if the computer is to do any more than mere mechanical work.

The use of the number of set-ups in a traverse line as the basis for the weight of line, in contrast to using the length of line, appears to be a desirable improvement. The writer often has used a weight which would fall about midway between the weight determined by length and by the number of set-ups. In this connection, it would appear that no harm would result from rounding off each weight value to the nearest single decimal place, although, of course, in the solution itself the terms must be extended to the proper number of decimals. The authors suggest that where level control lines are parallel to the traverse lines, corrections can be determined through one solution for both the traverse coordinates and the elevations. In such a case, the use of the number of instrument set-ups in each traverse line as the basis for its weight may not always prove to give the most logical weight for the parallel level line. Computers who face the situation of paralleling traverse and level lines should consider, thoroughly, this matter of best weights for each type of survey before combining the level and traverse adjustments in one solution. For instance, in adjusting levels, the weight for each line could be based, conceivably, on any one or any combination of the following: (1) The number of level-instrument set-ups; (2) the total difference of elevation (including both uphill and downhill); (3) the length of the line; (4) the average length of sight; or (5) (as suggested in the paper) the number of transit stations in the parallel traverse line.

The unique feature of the method described appears to be in the use of the unadjusted position or elevation value at each junction point, instead of in the attempt to determine first an approximately adjusted value which will lie quite near the finally adjusted results. Through this procedure, a considerable portion of the "assumed-minus-observed" terms becomes zero, thus greatly reducing the number of operations and making the adjustment more completely adaptable to machine computation. It is found that by using the adjustment diagrams as recommended by the authors, the adjustment equations of the "improved" method can be tabulated from the diagram immediately, thus eliminating the necessity for preparing the usual auxiliary and correlate tables of the conventional least-squares procedure.

It is found, as stated by the authors, that the mechanics of this improved method are very simple and can be grasped quickly even by freshmen cooperative students who have had no previous computing experience. The writers have adjusted or caused to be adjusted many hundreds of miles of both traverse and levels by using this "improved" method, and recommend it in every way.

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DISCUSSIONS

GENERAL WEDGE THEORY OF EARTH PRESSURE

Discussion

By Jacob Feld, M. Am. Soc. C. E.

Jacob Feld,[20] M. Am. Soc. C. E. (by letter).[20a]—So much has been written about the "wedge theory," as compared to the usual textbook presentation of the Coulomb theory, that a description of what Coulomb really did publish should be of interest. Most of the later descriptions of the Coulomb theory were apparently copied from either Woltmann's[21] translation (with errors contained therein) or from some later source originating from that translation of the original Coulomb's theory as published by him in 1773.[22] This was published in translation in *Böhms Magazine* in 1779, without change. Coulomb further developed his ideas in his "Théorie des Machines Simples," published in Paris about 1780, a second edition appearing in 1821. In the section on "Sur les Murs de Revêtements et l'Équilibre des Voûtes" (On Retaining Walls and the Balance of Arches), Coulomb covers the subjects of friction, cohesion, rupture of bodies, resistance of brickwork, pressure of earth, area and shape of rupture surfaces, arches without friction or cohesion, and equilibrium of arches, taking into consideration both friction and cohesion.

From the foregoing, it is seen that Coulomb merely treated the problem of earth pressure as a part of the general problem of evaluating friction and cohesion as resistances. It must also be kept in mind that Coulomb wrote before the invention of trigonometric symbols or ideas and computed all the ratios as algebraic fractions.

Coulomb lists preliminary propositions in the form of axioms as a basis for his theoretical development. Those which are of immediate interest in this discussion are:

Note.—This paper by Karl Terzaghi, M. Am. Soc. C. E., was published in October, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1940, by Howard F. Peckworth, M. Am. Soc. C. E.

[20] Cons. Engr., New York, N. Y.

[20a] Received by the Secretary January 10, 1940.

[21] "Beitraege zur Hydraulischen Architektur," by R. Woltmann, 1799 (IV Band), 1794 (III Band).

[22] "Essai sur une Application des Règles de Maximis et Minimis à Quelques Problèmes de Statique Relatives à l'Architecture," by C. A. Coulomb, Vol. VII, Royale Académie des Sciences, Paris, 1776 (1773 volume).

1. If a body is at rest on a horizontal plane, the resultant of the forces acting on it is normal to the plane and passes through the base of the body;

2. These forces can be counterbalanced by a single reaction equal to the resultant and acting opposite to it;

3. Resistance known as friction is proportional to the pressure; and

4. Cohesion is a measure of a resistance which a solid body opposes to disunion into two parts. In homogeneous masses, cohesion is proportional to the area of the section.

Coulomb then develops the value of the necessary resistance of a retaining wall with a vertical face supporting a level fill. The first development is for the assumption of a wedge sliding along a plane, along which there is friction and cohesion. An expression is developed for the active and passive lateral pressure acting normal to the vertical wall in terms of the height of wall, density of the fill, internal friction coefficient (the relationship to the natural slope assumed much later is not mentioned and never occurred in Coulomb's writings), the cohesion of the fill, and the surface width of the wedge of rupture.

In the next section, Coulomb discusses the possibility of a sliding prism along a curved surface, along which both friction and cohesion resistances are acting; but he states that experience (possibly "experiments") shows that the rupture is along a plane surface.

By the application of the theory of maxima and minima, Coulomb then develops an expression for the maximum value of the lateral pressures in terms of the friction and cohesion factors and shows that any cohesion reduces the values; therefore he assumes cohesion to be zero to obtain maximum results. The resulting formula, upon the introduction of the tangent of an angle as the friction coefficient, reduces to the well-known "Coulomb formula" which, for active earth pressure, is

$$P_A = m\,h_w^2 - c\,l\,h_w \qquad (5)$$

in which m and l are coefficients depending on the coefficient of friction, height of wall, and the density of the fill; h_w is the height of wall; and c is the cohesion coefficient. Where cohesion is not disregarded, Equation (5) is used for determining the minimum size of the retaining wall to balance moments about the heel.

By setting P_A equal to zero, Coulomb obtains the formula

$$h_e = \frac{c\,l}{m} \qquad (6)$$

as the height of excavation that will stand vertically unsupported because of cohesion.

The next section takes up the effect of surcharge, as an additional weight to that of the wedge of rupture, taking that part of the surcharge directly above the wedge.

The effect of wall friction is then evaluated by inserting in the general method a reduction of the weight of the wedge equal to the active pressure times the coefficient of friction of the fill against the wall. This coefficient is not assumed the same as the internal friction coefficient.

The final section starts to evaluate the area of the wedge of maximum pressure on an assumed curved surface of rupture, a part of which does not move because of cohesion. The mathematical detail became too involved, and the assumption is inserted: For the "fresh earth case"; that is, cohesion is zero, and a very involved formula is given for the shape of the curved surface of rupture. The basis of the derivation is the computation of maximum pressure additions of vertical earth sections of infinitesimal widths on vertical sections. The variable being the depth of the section,the resulting general equation gives the shape of the curve.

The writer always has been puzzled by the fact that no complete description of what Coulomb really contributed appears in the numerous texts on earth pressure, and it is hoped that its publication herein will correct many false impressions.

The author adds a description of a new thought to the solution of the problem—namely, the manner in which the wall or resistance tends to move in order to cause incipient failure of the fill. It was clear to Coulomb that no resistances would develop without such movement. Since the publication of this paper, there is little to add to the subject of earth pressures against rigid walls which move as a unit. However, the subject becomes infinitely more complicated when the attempt is made to apply the theory to flexible sheathing. In the writer's opinion, the field measurements of lateral pressure in back of sheathing are too indefinite to be of any value. The measurements are made on an indeterminate structure consisting of continuous sheathing supported by a series of wales and struts which were installed without regard to proper accurate location to avoid transfer of reactions. The reactions are a function of pressure intensity and location of the reactions. Slight changes in the bracing as installed change the reactions entirely. This is the explanation for the erratic and inconsistent curves in Fig. 9.

With reference to Fig. 9, there also seems to be a disregard of the reaction taken by the soil from the embedded portion of the sheathing (or soldier beams supporting the sheathing). Even with such omission, the total pressure included in area *A B C D* is 1.12 times the total theoretical pressure. The omission of the reaction at the bottom of the soldier beams explains the high location of the measured pressure, 0.53 h_e to 0.60 h_e, since those values are for the remainder of the pressure only. The reactions on certain struts were measured. It is not clear how these values were translated into pressures on the sheathing. It is hoped that the author will complete the missing steps in his closure.

In recent years so many papers have been published on various phases of soil mechanics which contain so little usable information that this clearly written description of the recent developments of the wedge theory comes as a pleasant surprise and a "landmark" contribution in the literature on lateral earth pressure. The history of the subject was published in 1928 by the writer.[23] The author fills in the copious additions to the subject since then.

[23] "History of the Development of Lateral Earth Pressure Theories," by Jacob Feld, *Proceedings*, Brooklyn Engineers' Club, January, 1928, pp. 61–104.

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DISCUSSIONS

RELATION OF THE STATISTICAL THEORY OF TURBULENCE TO HYDRAULICS

Discussion

By Messrs. Martin A. Mason, and J. C. Stevens

Martin A. Mason,[21] Assoc. M. Am. Soc. C. E. (by letter).[21a]—In an effort to clarify the present status of fluid turbulence research with respect to hydraulics, it seems pertinent to discuss briefly several concepts included in Professor Kalinske's paper, which may be sources of misunderstanding for the practical hydraulicians to whom the paper is addressed.

Unfortunately, much confusion of thought exists on the fundamentally important concept of the nature of turbulence. Among those thoroughly familiar with modern thought on the subject one finds turbulence as a generic term modified as isotropic, non-isotropic or anisotropic, initial, fully-developed, etc., each of which designations indicates to the experienced reader a characteristic phenomenon. To the practical man who is not familiar with the mathematical treatment of turbulence, however, these terms are usually meaningless. For these readers turbulence is a general designation for any or all types of irregular or agitated flow. Ordinarily flow in an open channel is "turbulent," as is the discharge over a dam spillway, the flow through a sluice gate, or the flow behind a bridge pier.

Obviously, before the hydraulic engineer can benefit greatly from the work of the investigators in the field of turbulence theory consistency of thought must be established as to the nature of turbulence. In this connection, Dryden[22] makes a pertinent comment when he writes, "It cannot be stated too emphatically that the definition of turbulence is necessarily arbitrary and the most useful definition depends on the purpose of the study." Thus, for the purpose of studying the theory of turbulent flow it may be convenient to define turbulence (as Professor Kalinske has done) as being of an entirely random nature without definite periodicity with time, since the mathematical treatment

Note.—This paper by A. A. Kalinske, Esq., was published in October, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1940, by Hunter Rouse, Assoc. M. Am. Soc. C. E.

[21] Asst. Hydr. Engr., National Hydr. Laboratory, National Bureau of Standards, Washington, D. C.

[21a] Received by the Secretary December 22, 1939.

[22] "Turbulence and the Boundary Layer," by H. L. Dryden, *Journal of the Aeronautical Sciences*, Vol. 6, No. 3, January 1939, p. 99.

is somewhat simpler for this case. In the case of flow along a boundary, which is the type most commonly encountered by hydraulicians, a more suitable definition of turbulence might be that it is motion such that correlation exists between the fluctuating velocity components at any point in the flow.

The translation of these statistical definitions into terms familiar to the average hydraulic engineer is beyond the scope of this discussion and the writer can emphasize only the importance of the suggestion that the reader not familiar with the concepts outlined consult certain works of Messrs. Dryden,[22, 23] Bakhmeteff, and Rouse.

If the foregoing definition of turbulence is admitted to be the most suitable for hydraulic engineering purposes, it is noted that the present statistical theory of turbulence does not offer immediate possibility of application to most hydraulic problems, inasmuch as it is chiefly concerned with isotropic turbulence. It must be admitted that, for the hydraulic engineer, at present, the statistical method of studying turbulence is only a useful tool which shows promise of being valuable under certain circumstances because of the simplifications it permits. Actually, at the moment, it is of little value for the solution of practical hydraulic problems, and hydraulic engineers should not expect it to supplant some of the older empirical methods of investigation. In this latter class one may cite the remarkable results achieved by L. Prandtl and his students, and by Theodor von Kármán.

Referring to Professor Kalinske's discussion of energy dissipation, it is suggested that, from the hydraulician's point of view, the determination of the time period necessary for the required dissipation of energy in the form of anisotropic turbulence is perhaps more important than the determination of the rate of dissipation. For example, consider the case of a dam spillway; the kinetic energy of the discharging water may be changed rapidly, and in substantial part, to anisotropic turbulent energy in the stilling basin; but, unless this turbulent energy is substantially dissipated in the basin, scour of the downstream bed will result. As shown by Professor Kalinske, the determination of the rate of dissipation of turbulent energy is rather complicated even for the relatively simple case of isotropic turbulence, a condition seldom, if ever, encountered in practice. It would seem certain, however, that (as indicated by the interpretation of the term λ as a measure of the diameters of the smallest eddies responsible for the dissipation of energy[6]) the smaller the size of the eddies the more rapid will be the dissipation of energy, and consequently the smaller the required basin length.

As suggested by Professor Kalinske in "Miscellaneous Problems" (No. (5)), model studies involving energy dissipation, sediment transportation, or bed-load movement require that dynamic similarity exist between the intensity and scale of the turbulence in model and prototype. A somewhat related consideration is found in wind tunnel studies where a similarity of turbulence in the tunnel is necessary to obtain comparable results from different wind tunnels. This condition led Dryden to remark[22]

[23] "Turbulence, Companion of Reynolds Number," by H. L. Dryden, *Journal of the Aeronautical Sciences 1*, No. 2, 67, April, 1934.

[6] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 421.

> "* * * the turbulence characteristics of the wind tunnel need to be known and taken into account in the interpretation of measurements. * * * We have to do essentially with a new independent variable whose effects are as important as those of Reynolds number and in many ways similar to those of Reynolds number * * *."

With this latter thought in mind, one may derive from the Navier-Stokes equations (which are applicable to turbulent flow since the velocity components used in the equations refer to instantaneous velocities) not only the usual parameters (Reynolds, Froude) governing the relation of model to prototype; but also, a third parameter having a form similar to Froude's number, which governs the viscous stresses in the two structures. The thought is suggested that this third parameter might be of the nature of a turbulence parameter, whose function is to relate the turbulence in model and prototype.

It should be remembered that similar Reynolds numbers for model and prototype do not alone guarantee similar conditions of turbulence unless there is complete geometric similarity. This may be shown as follows: The cause of the discrepancy in turbulence values in wind tunnels is the effect of the grids or vanes in the tunnel, the turbulence in the tunnel being a characteristic of the grid or vane installation. The corresponding source of discrepancy in hydraulic models would be the stilling racks or basin at the water supply end of the model. If such devices are not similar to those in the prototype, or do not establish flow conditions comparable to those in the prototype, there is no assurance that the intensity and scale of the turbulence in model and prototype are similar even if the same Reynolds number obtains for the two.

The possibility of the existence of a scale effect of turbulence between model and prototype would appear to be sufficiently real to justify the attention of model experimenters in those cases where the turbulence characteristics of the water are important.

Regarding experimental techniques, the writer had the opportunity to develop a method of observation of a stream injected into flowing water which uses fluorescent products illuminated by ultra-violet light.[18] The method shows some promise but has not yet been developed as fully as the excellent technique used by Professor Kalinske. Its principal advantage lies in the fact that the injected stream is seen as a lighted streak on a dark background, thus avoiding the interference of parasitic light reflected from the impalpable particles carried by the water.

In any photographic method employing foreign particles to reveal the motion of the water some uncertainty attaches to the assumption that the added particles truly follow the exact motion of the water particles. This would be true especially when the added particles are of the nature of droplets of an immiscible liquid, unless the droplets are exceedingly small. Similarly, the use of an injected stream of color may be questioned because of possible inertial and viscosity effects. However, Charles Camichel and his co-workers have shown[24] that aluminum particles approximately 0.1 mm in diameter,

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[18] "Contribution à l'Étude de la Mesure des Débits d'Eau par la Méthode Allen," by Martin A. Mason, Doctorate Thesis, Univ. of Grenoble, France.

[24] "Sur le régime permanent dans les chambres d'eau," by Charles Camichel, *Revue Générale d'Electricité*, Vol. 8, p. 331.

when washed in alcohol and introduced into a body of water, reproduce exactly the motions of the liquid particles they replace. It would be interesting to compare the magnitude of the droplets used by Professor Kalinske with the size of the particles recommended by M. Camichel.

A photographic method which has been used with considerable success in France for the study of the instantaneous distribution of velocities around a propeller would seem to offer great possibilities for the study of the turbulence problem. First suggested by Marey,[25] developed by numerous investigators, particularly M. Camichel,[26] and in 1937 completely described by M. Chartier,[27] the method is of particular interest because of its applicability to the study of any type of fluid motion in space by stereoscopic means.

In brief, motion pictures are taken of the movement of entrained aluminum particles, utilizing stroboscopic light. A comparison stereopticon of special design is then used in the study of each frame of the film obtained, or the motion may be studied by ordinary projection means. By the use of motion pictures combined with stroboscopic illumination the paths of the illuminated particles over very short time intervals are obtained as a series of dashed streaks, from which the particle velocities may be computed. The minimum interval of time recommended for the stroboscopic illumination is of the order of $\frac{1}{800}$ sec. An improvement of the method, resulting in a considerable saving of time, is the use of a stereo-projector so arranged as to locate the particle streaks on a coordinate system. This arrangement permits the determination of velocities within about 1% error, and the determination of position in space with about 5% error.

J. C. Stevens,[28] M. Am. Soc. C. E. (by letter).[28a]—Professor Kalinske has been continuing the research project "Conversion of Kinetic to Potential Energy in Expanding Conduits" started by Frederic T. Mavis, M. Am. Soc. C. E., for the Special Committee of the Society on Hydraulic Research.[29] The basic method of approach to this problem has been well set forth in this paper. The author has brought the practical aspects of fluid turbulence home to the hydraulic engineer.

Lately hydraulic engineers have recognized that the energy gradient in tortuous conduits is quite uniform and that energy losses are not concentrated at bends as was long believed. In the light of the author's treatment the explanation is simple: The so-called losses are merely heat dissipation through the viscous properties of water in turbulent motion that continue downstream far beyond the origin of the turbulence. In uniform flow the boundary contacts are a source of continuing turbulence, a balance having been reached between that and heat dissipation. Bends, expansions, and piers are sources

[25] "Le Mouvement des liquides étudié par la chronophotographie," by E. J. Marey, *Comptes Rendus de l'Académie des Sciences*, May 9, 1893, Vol. CXVI.

[26] "Notice sur les travaux de M. Camichel," November 15, 1929, p. 21.

[27] "Chronophotogrammétrie plane et stéréoscopique," by Charles Chartier, *Bulletin No. 114*, Publications Scientifique et Technique du Ministère de L'Air, 1937.

[28] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[28a] Received by the Secretary January 15, 1940.

[29] *Civil Engineering*, March, 1938, p. 195.

of concentrated turbulence, and the heat dissipation from them persists for a long way downstream, thus giving a more or less uniform gradient of energy losses.

The author states (see paragraphs following Equation (21)): "* * * the smaller the eddies the higher the rate of energy dissipation will be." Perhaps this explains why the hydraulic jump is such a potent source of energy dissipation, most of the energy being lost in a comparatively short reach. This must be due to the violent internal eddies set up as a result of the sudden expansion, the visible, foamy, front roller being responsible for only a relatively insignificant portion of the total loss.

The data in Figs. 7 and 8 are of particular significance in explaining why kinetic energy cannot be converted to potential energy without inherent losses. As the author states (see heading: "Turbulence and Energy Considerations"), turbulent energy is a degraded form and is irrecoverable, thus being in broad conformity to the second law of thermodynamics. If, in Fig. 8, the water were caused to flow in the opposite direction, it is likely that the $\bar{u}^2$-curve and the $\bar{v}^2$-curve would become even less prominent than they appear in the uniform pipe of Fig. 7. This feature may be worthy of further investigation.

There is need of means and a method of measuring velocities at any point in a flowing prism without inserting something that itself causes turbulence. Professor Kalinske has solved the problem very cleverly by the use of a motion-picture camera and droplets of an immiscible liquid of the same specific gravity as water. These can be injected into the prism far enough upstream so that the slight turbulence caused by the needle becomes negligible at the camera.

The tedious labor involved in analyzing the data thus gathered, however, is a serious handicap to the method. Perhaps simpler, yet just as effective, means will be found in time.

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DISCUSSIONS

EFFECTIVE MOMENT OF INERTIA OF A RIVETED PLATE GIRDER

Discussion

BY MESSRS. ALVIN B. AUERBACH, JONATHAN JONES, AND R. L. MOORE

ALVIN B. AUERBACH,[23] JUN. AM. SOC. C. E. (by letter).[23a]—To paraphrase an utterance of Hardy Cross, M. Am. Soc. C. E., a structure, not being cognizant of its designer's assumptions, always deforms in accordance to the laws of geometry and statics.[24] Both common design practice and experience agree that the deflections of a girder are a function of the gross section. Hence a suitable design method should:

1. Be based upon the deformation of the girder and therefore on the gross section;
2. Yield maximum computed stresses in consonance with the underlying assumptions of the specifications governing the design;
3. Be capable of being visualized by the user; and
4. Be adaptable to girders of unusual shape or section.

Under "Conclusions" the authors state: "It is believed that the data from these tests point to the acceptability of the gross moment of inertia for design purposes." The writer proposes to show that it is possible to analyze a structure so as to yield higher design stresses which are still in consonance with the average measured stress found by the authors.

For example, a method of design which meets Conditions 1 to 4 is the one utilizing the ratio of the equivalent net flange area to the gross. It was used in the U. S. Engineer Office, Rock Island (Ill.) District, for the design of the plate girders having a concave lower flange, and unsymmetrical sections. These girders were the service bridge spanning the movable gate openings on the

NOTE.—This paper by Scott B. Lilly, M. Am. Soc. C. E., and Samuel T. Carpenter, Assoc. M. Am. Soc. C. E., was published in October, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1939, by Messrs. William R. Osgood, Clyde T. Morris, B. R. Leffler, E. Neil W. Lane, Lewis E. Moore, W. E. Black, and L. E. Grinter; and January, 1940, by Messrs. Charles M. Spofford, E. Mirabelli, Edward Godfrey, Walter H. Weiskopf, and C. D. Williams.

[23] 1st Lieut., Corps of Engrs., U. S. Army, Fort Belvoir, Va.

[23a] Received by the Secretary December 26, 1939.

[24] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1373.

Mississippi River dams. Using the relationship derived in standard texts on design of plate girders, the equivalent gross area is expressed by:

$$A_e = A_f + \frac{A_w}{6} \quad \text{.........................(7a)}$$

and the equivalent net area by:

$$A_e' = A_f' + \frac{A_w}{\frac{6\,p}{p - r}} \quad \text{.........................(7b)}$$

in which A_f and A_f' are the gross and net flange areas, respectively; A_w is the gross area of the web; p is the vertical pitch of rivets in the web (as in stiffeners); and r is the diameter of the rivet holes.

Accordingly, in compression,

$$s = \frac{M\,c}{I_{(\text{gross})}} \quad \text{.........................(8a)}$$

and, in tension,

$$s = \frac{M\,c'}{I_{(\text{gross})}} \times \frac{A_e}{A_e'} \quad \text{.........................(8b)}$$

Using the data supplied by the authors, for girder D-3, the areas are as shown in

TABLE 8.—EQUIVALENT FLANGE AREAS, IN SQUARE INCHES

Section	Dimensions, in inches	EQUIVALENT AREAS	
		Gross, A	Net, A'
Web	12 by 5/16	0.63	0.63
Angles	2½ by 2½ by ¼	2.38	2.04
Cover plate	6 by ¼	1.50	1.16
Cover plate	8 by ¼	2.00	1.66
Totals	....	6.51	5.49

Table 8. From Column (9), Table 3, the computed stress for the gross section is 19.1 kips per sq in.; the maximum tensile stress would be $19.1 \times \frac{6.51}{5.49} = 22.7$ kips per sq in.; and thus the average tensile stress $\frac{22.7 + 19.1}{2} = 20.9$ kips per sq in. This is in fairly good agreement with the measured average tensile stress of 20.2 kips per sq in.

The results obtained by this method, although numerically slightly higher than other methods, better meet the desired conditions. The underlying reasoning is simple; it is easily adaptable to unusual girders. Also it does not encroach on the much abused factor of safety, which, already carrying allowances for defects of material, imperfections of workmanship, secondary stresses, local stress concentrations, and other intangibles, needs some protection lest it become the great unknown. It may be interesting to note that in military design of bridges in war time the factor of safety based on existing civil practice is to be flatly reduced by 20% for steel structures.

Most present-day specifications utilize design stresses based on double net areas, with results that are almost identical with the foregoing; but these specifications are not feasible for unsymmetrical girder sections. If stresses are to be raised, let the design stresses be increased openly rather than permit the increase to be hidden in illogical analysis.

JONATHAN JONES,[25] M. AM. SOC. C. E. (by letter).[25a]—When the Committee on Specifications of the American Institute of Steel Construction determined (1936) to adopt gross moment of inertia for the design of plate girders in building construction, it did so on the basis of a reasoning, satisfactory to its members, with respect to actual unit stresses on sections through rivet holes. That reasoning may be summarized somewhat as follows:

(1) A statically-loaded plate girder can scarcely be made to fail in the tension flange; the compression flange and the web usually determine the capacity.

(2) All manner of things have happened to overloaded crane girders in mill buildings, but not to the tension flanges.

(3) In normal, simple-span girders the flange area removed by rivet holes is less than 15%. The rivets are practically always machine-driven, and they induce a friction in the heads which carries part of the stress.

(4) If occasional rivets are loose, and the friction in Item 3 is nonexistent, the most severe stress that can occur is somewhat less than 23,000 lb per sq in., and the designer would rather have 23,000 in a plate girder flange than 20,000 in many other elements of a building.

(5) Flanges that are "chopped up" by closer spacing of rivets, on account of heavy shears to be developed in short distances, should not be required to carry still greater net stresses due to excessive reduction in area; and therefore a maximum exempted reduction of 15% resulting from a "chain" of holes was established.

(6) Holes filled with bolts, pins, or countersunk rivets have less hope of being assisted by friction, and should be deducted.

The specification clause[26] was drafted to authorize the use of gross sections, subject to the foregoing implied limitations. It was submitted to a number of engineers and met, not with unanimous, but with general, approval.

The desire for a research project arose after the specification had been thus drafted and adopted; its purpose was not to convince the Specification Committee, which had already acted, but to reconcile, if possible, some objectors to what the Specification Committee had done. No one expected to show, by a research project, that there are not peak points of high unit stress around rivet holes; or that, if a tension flange could be made to rupture, it would rupture elsewhere than through a hole. What could be shown, and has been shown, is that

(*a*) Exemption of rivet hole deductions to as much as 15% of the gross flange area is conservative (rivet hole deduction in test specimens 18.0% minimum and 23.4% maximum);

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[25] Chf. Engr., Fabricated Steel Constr., Bethelhem Steel Co., Bethlehem, Pa.

[25a] Received by the Secretary January 5, 1940.

[26] A.I.S.C. Specification for Design, Fabrication and Erection of Structural Steel for Buildings, Section 19*a*, 1936; also A.I.S.C. Manual on "Steel Construction," Third Edition, October, 1937, p. 272.

(*b*) Drilling holes in a solid flange increases the average unit stress over that in the solid material; but

(*c*) Bolts or rivets in these holes will go a long way toward reducing this average unit stress to what it was in the solid material;

(*d*) Bolts performed about as well as rivets in this respect, but these were tightened, of course, with considerable care.

The research project has been successful in demonstrating all that was hoped for. It has not disposed of the spots of overstress backed up by lower-stressed material, which were not the subject of this particular study. Engineers still may be divided into those who believe that the gross-section-modulus specification represents a sound attitude, and those who believe otherwise. For the former group, including the A.I.S.C. Specification Committee, it may be said that, as raw materials tend toward scarcity and high cost, it will be more and more necessary to dispense with inherited extravagances which a practical consideration of probable behavior may serve to eliminate.

R. L. MOORE,[27] ESQ. (by letter).[27a]—Certain phases of plate-girder behavior have always been open to question, namely: (1) What are the effects of rivet holes upon the behavior of a plate girder; and (2) what allowance should be made for their effects in design? As is generally recognized, specifications are not in agreement on the latter question. In some cases plate girders are proportioned on the basis of the gross section;[28] in others the net section of both tension and compression flanges is used in computing tensile stresses, whereas the corresponding compressive stresses are based on the gross section.[29]

Apparently, the logic behind the use of net sections in proportioning plate girders is that such a procedure provides some allowance for a reduction in strength resulting from rivet holes in the flanges. Little or no information, either from experience or tests, is available, however, to indicate how essential such a procedure may be. It is generally assumed, as stated in the paper, that bending deflections may be computed with reasonable accuracy, using moments of inertia based on gross sections. This view is supported by Table 3 in which the differences between observed effective moments of inertia and those computed on the basis of gross sections averaged only 3% in the case of specimen PT-1 and 6% for specimen PT-2, with the maximum individual differences amounting to only about double the foregoing percentages in each case. If moments of inertia for gross sections are satisfactory for computing deflections, it would seem to follow that they are also satisfactory for computing average flange stresses, since deflections are merely a reflection of the strains corresponding to such average stresses. The computation of stresses on the basis of net section may result in increases of from 10% to 15% over those obtained using gross sections; but such increases are of little significance in view of the stress concentrations of from two to three times the average actually produced in the flanges by the rivet holes. As long as questions concerning fatigue strength

[27] Research Structural Engr., Aluminum Research Laboratories, A.C.O.A., New Kensington, Pa.

[27a] Received by the Secretary January 8, 1940.

[28] See A.I.S.C. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, Section 19*a* (revised 1936).

[29] See A.R.E.A. Specifications for Steel Railway Bridges, Art. 426, 1938.

under repeated loadings are not involved, and average rather than maximum values of flange stress are used as a basis for design, the use of gross sections seems satisfactory.

The authors state that their tests were planned to discover, if possible, how much the flange stresses in plate girders are affected by the rivet holes in the flanges. Unfortunately, they have presented no data pertaining to the actual stress distribution in the flanges; neither have they given any information to indicate what effect the stress concentrations at the rivet holes might have on the ultimate strength of the girders. The fact that some of the 10-in. gage lines used in their tests were located in line with the holes in the flanges means that the changes in length measured included hole distortions as well as strains in the material. Obviously, such deformations have no bearing upon the actual stress distribution, although they would be expected to be consistent with the strains corresponding to the measured deflections.

The observed stresses shown in Table 3 are in good agreement with the computed values based on either the observed effective moments of inertia or those based on gross sections, since there is little difference between these quantities. Although the stresses measured in the tension flanges were consistently higher than those found in the compression flanges, the agreement between measured and computed values based on net sections was not as satisfactory as that found for gross sections. The discrepancy would have been even greater if the authors had made reductions in area from both tension and compression flanges in computing net sections, which is the procedure suggested by the measurements made of the position of the neutral axis. In compression flanges, where the holes are filled tightly by rivets or by body-bound bolts, there may be some justification for assuming these flanges to be solid and making deductions from the tension side only. Where there are open holes in both tension and compression flanges, however (as in the authors' series B tests), or where the holes are filled with loose fitting bolts (as in the series C tests), there would seem to be no reason for not making deductions in area from both flanges if the question of net section is really a significant one.

In order to corroborate the authors' general conclusion that the gross-section method is satisfactory for computing plate-girder stresses in most cases, a few experimental data from a test on an aluminum-alloy plate girder have been included in this discussion. Fig. 9 shows the measured stress distribution in the extreme fibers of the web of a girder section subjected to pure bending. The stresses shown correspond to strains obtained by means of tensometers on ½-in. gage lengths midway between, and directly over, the flange rivets. The interesting fact about these data is that the stresses between rivets are higher than on the net section through the center of the rivets. Although this behavior is consistent with the elastic theory regarding the effect of stress concentrations produced by holes near the edge of a wide plate, it is contrary to the result expected when the net-section method is used to determine stresses in design. The stresses measured on the tension side of the web were consistently higher than those found on the compression side (as was also found in the authors' tests), but the maximum values in both cases are reasonably consistent with the stresses computed on the basis of the gross

section. The use of the net section for the tension flange would increase the computed stresses in that flange by about 11%, but would give a greater difference between measured and computed stresses than that indicated in Fig. 9.

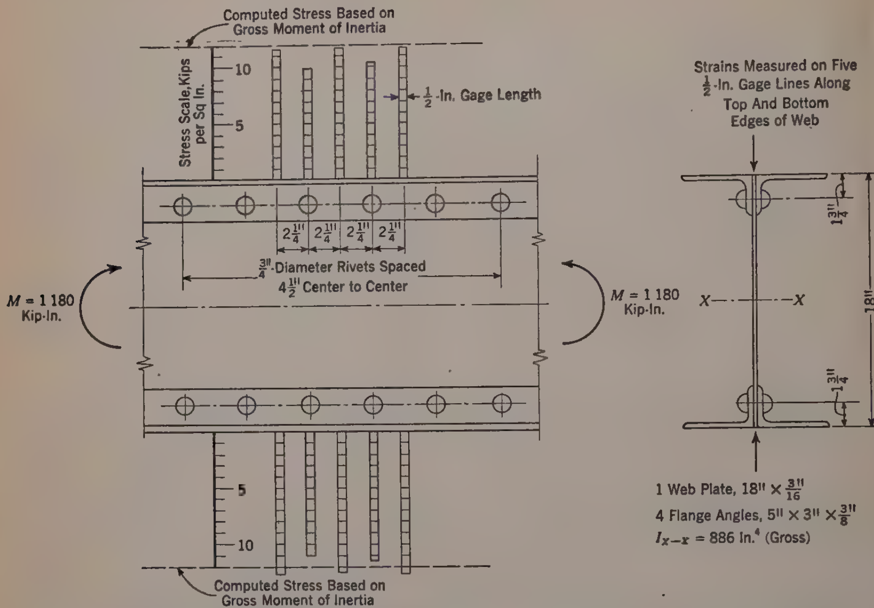

FIG. 9.—MEASURED STRESS DISTRIBUTION IN WEB OF ALUMINUM ALLOY PLATE-GIRDER SECTION SUBJECTED TO PURE BENDING

The authors propose a formula for computing effective moments of inertia of plate girders which involves not only the rivet pitch and hole diameter, but an experimentally determined constant which they have evaluated within certain limits. Such a refinement in the computation of stresses and deflections scarcely seems justified from the data given in the paper. One objection that might be raised to such a procedure is that no consideration is given to the possible lack of integral action between the elements of a plate girder. For some proportions it appears that this factor may be of even more significance than the effect of rivet holes. The various parts of a plate girder function, of course, through their connections. Load can be transmitted from the web to the flange angles and cover plates only through the action of the rivets, which involves deformations and slip. It would have been of interest in this connection if the authors had included some of the strains measured in the web and flanges of the test girders. The two types of data did not indicate the same position of the neutral axis, as shown in Table 1. It is conceivable that they might also have thrown some light on the question of integral action.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE RÔLE OF THE ENGINEER IN AIR SANITATION A SYMPOSIUM

Discussion

By Messrs. Theodore Hatch, Donald Francis Griffin, Gordon M. Fair, and H. G. Dyktor

Theodore Hatch,[18] Esq. (by letter).[18a]—Mr. Bloomfield has indicated, in a broad way, some of the problems that confront the engineer in connection with the maintenance of health in industry and has illustrated these problems by a detailed discussion of the methods of analysis employed in the study of one particular health hazard—namely, mercury poisoning in connection with the preparation of hatter's fur. It should be pointed out that the procedures employed today in studies of this kind have been developed to a very large extent by Mr. Bloomfield and his associates in the U. S. Public Health Service.

To the sanitary engineer there is nothing basically new in the problems of industrial hygiene and sanitation since the objectives here, as in other phases of public health engineering, are to discover the nature of the relationship between health and environmental factors, to determine optimum values for health and safety with respect to hazardous conditions, and to develop control measures that will insure the maintenance of a healthful environment. The close cooperation between the physician and the engineer is essential in connection with this as well as other public health problems.

Sanitary engineers will also discover nothing basically new in the approach to the study of sanitation in industry as outlined by Mr. Bloomfield. The detailed techniques described by him, however, are probably not familiar to most sanitary engineers. He has limited his paper to one very small industry; but in doing so he emphasizes the great variety of the health problems that one encounters in industry and the corresponding variety of instruments and methods of analysis required for their study. He has emphasized further,

Note.—This Symposium was published in November, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the papers.

[18] Associate Dust Control Engr., Div. of Industrial Hygiene, New York State Dept. of Labor, New York, N. Y.

[18a] Received by the Secretary December 2, 1939.

however, that in all these problems the fundamental objectives in the investigation remain the same.

Work in industrial hygiene and sanitation in the United States extends over a period of more than 30 yr, but the most rapid expansion in this field has occurred in the last decade. In the beginning, the need for working tools tended to emphasize instruments, analytical methods, and laboratory experiments, but recently there has been a great increase in the number of governmental bureaus of industrial hygiene whose first interest must be to develop and maintain health in industry and not to develop instruments and methods of analysis. Thus, the practical exigencies of their duties may make it necessary for the workers in such bureaus to sacrifice elaborate scientific methods for procedures that will yield quick results. One of the best illustrations of this is seen in the problem of establishing standards of atmospheric cleanliness.

From the standpoint of the laboratory investigator, the best way to determine the toxic limit of an organic solvent or other industrial poison is by means of controlled animal and human experiments, so designed as to permit the complete exploration of the many different aspects of the problem. Considerable work has been done in this connection, but in practice one cannot depend upon costly and time-consuming experiments to supply the needed information with respect to the great variety of problems encountered in industry. Another approach to this problem is largely of an engineering nature and consists in determining the level of concentration of the toxic substance that can be secured by means of the best methods of control available to the industry and to fix upon this as the criterion of good operation. As control measures improve in efficiency, the permissible concentration would be lowered. There is precedent for this in all fields of public health engineering. The so-called Treasury Standard for public water supplies, for instance, establishes bacterial and chemical concentrations based almost entirely on the good practice in water purification rather than any direct epidemiological correlation between water pollution and disease. In the field of industrial sanitation this method has been employed with spectacular success in the gold mines in South Africa.

With the development of the konimeter in 1916, dust surveys showed that the best practices in dust control reduced the mine dust concentration to 300 particles per cubic centimeter, and this value was adopted as a "figure of merit" or criterion of dust control for all mines. Since that time, the progressive decrease in dustiness has been followed by a corresponding decrease in the incidence of silicosis, and present experience indicates that the annual incidence of this disease has been reduced to a fraction of 1%. At the same time the average dust concentration has been reduced to below the standard established in 1916.

This engineering approach is probably the only one possible in certain industries. In foundries, for example, it is doubtful if one can show a direct relation between the degree of dust exposure, as measured by a dust survey, and the degree of lung damage, as measured by chest X-ray pictures and other examinations. This does not mean that a relationship does not exist but that

it is overshadowed by other and more important factors, such as individual susceptibility. Moreover, the prevention of silicosis does not constitute the principal reason for dust control in the foundry industry. Foundry workers exhibit a higher mortality from other respiratory diseases, and dust control becomes a more general health measure. One may say, therefore, that the dust concentration in this industry should be maintained at the lowest possible level consistent with good engineering practice.

The conditions of sanitation and hygiene existing in industry have considerable influence upon the health record of the nation since nearly 10 million workers are exposed to the influence of their industrial environment for a third of each day. Industrial hygiene concerns itself, therefore (as Mr. Bloomfield states), with more than the control of specific poisons, which may be spectacular but not of great quantitative significance. Thus, silicosis and asbestosis are not widespread industrial diseases, but, on the other hand, the health of industrial workers is influenced to a very high degree by industrial dust in general. One of the important objectives of industrial sanitation is to introduce adequate measures of dust control into all dust producing establishments, regardless of the chemical composition of the dust. In the same way, problems of general ventilation, noise control, fatigue in relation to the environment, general plant sanitation, housing of the industrial worker, the effects of mass production, etc., all affect the health of the worker and should come within the purview of the engineer in this phase of public health work.

In conclusion, one may agree with Mr. Bloomfield "that the field of industrial sanitation is not only virgin but will continue to be an interesting subject for many years to come—one which will test the ingenuity and ability of the engineering profession."

Donald Francis Griffin,[19] Jun. Am. Soc. C. E. (by letter).[19a]—One source of a particularly obnoxious type of air pollution is the emission of hydrogen sulfide gas by decomposing sewage. The metropolitan area along the east San Francisco Bay in California is a notable example of a community whose air is being polluted from this source. In addition to the exposure to this type of pollution of a large residential area adjacent to a 20-mile waterfront, along which outfall sewers discharge, approximately 26,000,000 automobile passengers were exposed to it along the East Shore Highway during 1939.

There is the further possibility that the air, at least in the vicinity of the East Shore Highway, may be polluted with pathogenic organisms commonly found in raw or untreated sewage, through a combination of wind and wave action wafting droplets of infected water into the air.

The number of bacteria contained in such droplets "* * * depends upon the kind, the source, the conditions which lift them into the air, their survival rate, the air currents which transport, disperse and dilute them within the atmosphere, the rate of deposition and the physical and chemical conditions of their atmospheric environment."[20]

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[19] Associate Planning Technician, National Resources Planning Board, Berkeley, Calif.

[19a] Received by the Secretary December 26, 1939.

[20] "Preventive Medicine and Hygiene," by M. J. Rosenau, Ed. 6, 1935, p. 909.

Very small droplets may evaporate before the force of gravity can precipitate them from the air; thus droplet or water-borne infection may change into air-borne infection. Under such circumstances short-lived pathogenic organisms may become air-borne.

In a test to determine the viability of droplet nuclei infection "* * * none of the four organisms typical of the intestinal tract, B. coli, B. dysenteriae Hiss Y, B. typhosus and B. paratyphosus A, were recovered at the end of the first day or at the end of eight hours when samples were taken after this time interval."[21] However, the interval between the time a droplet infected by one of the aforementioned organisms is wafted into the air and the time the infection becomes air-borne may be considerably less than eight hours. Perhaps additional tests using shorter time intervals are necessary to determine the viability of these organisms. This would appear especially desirable where large numbers of people are exposed near a source of droplet infection, as in the example of the East Shore Highway.

Assuming that infection may occur from such apparently short-lived pathogenic organisms as those mentioned, when air-borne, factors such as the number of people, and their proximity to sources of droplet infection, become important along with the viability of droplet nuclei infection. Present and future investigations may disclose that bacteria-infected air should be considered in relation to other factors as well as with contamination by organisms of the human respiratory tract.

GORDON M. FAIR,[22] M. AM. SOC. C. E. (by letter).[22a]—Professor Phelps has given a stimulating and comprehensive survey of air sanitation, a field of engineering endeavor which, in its relation to the preservation and promotion of public health, has been gaining in recognized importance during the last few years. The future of air sanitation seems most promising. The sanitary control of the air, as outlined by Professor Phelps, offers broad and varied opportunities for engineering research and practice. In many respects it remains one of the untilled fields of public health. The engineer is certain to play an important rôle in the tillage of this field, provided he is willing to depart from classical engineering disciplines in a manner similar to the departure from routine civil engineering by the founders of water sanitation.

Three factors in air sanitation are of sufficiently broad significance to be added to those included in Professor Phelps' survey: (1) The control of odors; (2) the conditioning of air under abnormal pressures; and (3) the modification of the illumination of enclosed spaces.

Pollution of the outdoor atmosphere by odors has its source in industrial activities, including the treatment and disposal of sewage, industrial wastes, garbage, and refuse, as well as in other human activities such as the combustion of fuel for heating and transportation. The air of confined, occupied spaces may be rendered odoriferous by industrial and household processes and by the normal physiological processes of the occupants. Although odors are not known

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[21] "Air-Borne Infection," by W. F. Wells and M. W. Wells, *Journal*, Am. Medical Assoc., 107, November 21, 1936, p. 1700.

[22] Gordon McKay Prof. of San. Eng., Harvard Graduate School of Eng., Harvard Univ., Cambridge, Mass.

[22a] Received by the Secretary January 5, 1940.

to be harmful, they do create discomfort and are important factors in air sanitation. The control of body odors may indeed be one of the limiting conditions in the ventilation requirements of closely occupied spaces such as school rooms, public vehicles, and assembly halls. Under the stimulus of odor control in water supplies, something has been learned about the measurement of odors, but much remains to be discovered about the differentiation of odors and their removal from the atmosphere by means other than their dilution to threshold values. The physiological and pathological response to odors also requires further investigation.

The return from high pressures to normal atmospheric conditions is frequently accompanied by disorders known as compressed-air illness, or caisson disease. These disorders are caused by the liberation of free nitrogen into the tissues and blood and the formation of bubbles of the gas which may block circulation. Similar phenomena may be associated with the entrance into low pressures from normal atmospheric conditions. High pressures are generally associated with industrial occupations such as diving, the sinking of caissons or shafts, and tunneling. Low pressures may be encountered in high-altitude flying and possibly in the treatment of certain pulmonary diseases. In each instance, the change that produces the disorder may be tempered by suitable engineering means. There is also a possibility of modifying the quality of the air itself within the abnormal atmosphere.

Although the control of the indoor air will probably remain very largely a matter of adjusting the temperature, humidity, and movement of the air and of cleansing the air supply, it is conceivable that sources of light, more closely akin in their radiations to sunlight and perhaps even more abundant in selected wave lengths, may play a part in the newer air sanitation. Such radiations are not generally emitted by present-day sources of illumination. Professor Phelps has mentioned the use of ultra-violet light for the disinfection of air but has not considered the wider application of beneficent radiations.

He ends his survey with the statement that "it is to the public health engineer that the future must look for guidance and responsible control" in air sanitation. This is a thought-provoking conclusion of importance to members of the Sanitary Engineering Division of the Society, as well as to sanitary engineers in general. It naturally raises the question as to whether it is practicable for sanitary engineers to encompass all of the branches of sanitation. Will they become "masters of none" if they make the attempt, and will they obstruct the wider usefulness of engineers in public health work? (The writer is unable to differentiate between "sanitary engineering" and "public health engineering." As Lemuel Shattuck's Sanitary Commission of Massachusetts stated in 1850: "The word 'sanitary' means 'relating to health.' " This sentence is amplified in a footnote of the Commission's Report from which the following may well be repeated. "We would divest our subject of all mystery and professional technicalities; and as it concerns everybody, we would adapt it to universal comprehension, and universal application." Has the word "sanitary" acquired a different connotation since Lemuel Shattuck's day?)

As previously stated, air sanitation as a field for study and professional activity has come to the front within the last few years. The training of many

state and municipal sanitary engineers in this branch of sanitation appears to have been inadequate, as evidenced by the fact that the U. S. Public Health Service has found it expedient to conduct short courses in industrial hygiene for such engineers. The writer knows of only a few universities in which rigorous training in air sanitation is available to engineering students. As stated by Professor Phelps, engineering design connected with air sanitation has been done in the past by groups of engineers who, however willing to meet requirements that health authorities may have prescribed, have lacked the background of "knowledge of the physical, chemical, bacteriologic, physiologic and toxicologic properties of the special types of impurities" on which the sanitary control of the air must be founded. Basic research by engineers appears to have been limited or spasmodic.

The group of engineers that has been widely interested in emphasizing the public health aspects of certain of its engineering activities has been the civil engineering group. This is attested by the existence of the Sanitary Engineering Division in the Society. It follows that, in the past, most sanitary engineers have dealt more particularly with water sanitation. This situation is natural, since opportunities for the application of specialized public health knowledge have been most favorable in engineering endeavor connected with water supply and sewerage. The incentive for study, in a practical world, must be provided of necessity by the opportunity for gainful occupation.

It has been the writer's experience that, in the normal period of study commonly available to the training of sanitary engineers, it is not possible to include too large a group of disciplines without sacrificing thoroughness. There are certain fundamental disciplines, lying outside the realm of engineering proper, that are not shared alike by the engineer who is to fit himself for the sanitation of water and the engineer who is to become competent in the sanitation of air. Notable among these disciplines is human physiology, or ecology, coupled with toxicology; these are subjects essential to the intelligent and intellectual appreciation of air sanitation, but not so important in water sanitation. Furthermore, there must be a difference in emphasis upon certain basic engineering sciences, as well as biological and chemical sciences, in their relation to water on the one hand and air on the other.

Given sufficient time for study, it is possible to train men to become qualified in both water sanitation and air sanitation. The writer does not believe, however, that in the ordinary management of human affairs an adequate number of men can hope to maintain enduring intellectual and practical contacts with both these branches of sanitary engineering. In the long run, expertness in one or the other branch must be sacrificed if degeneration in both is to be avoided. It would seem wise, therefore, to recognize that a diversity of interest and practice in sanitary engineering is a norm of the profession.

Only two branches of sanitation have been contrasted—namely, water sanitation and air sanitation. There are others: Milk, shellfish, and food sanitation; insect and rodent control; housing; illumination; noise suppression; and the disposal of solid municipal refuse. It stands to reason that engineers whose main interest lies in water supply and sewerage will continue to be recruited chiefly from the ranks of civil engineers and chemical engineers, whereas

engineers who deal more particularly with the sanitation of the air may more probably possess a background of mechanical or chemical engineering. Civil, mechanical, electrical, architectural, and agricultural engineers may share in other branches of sanitation. Sanitation, indeed, must enter in some measure into all engineering practice because engineering is essentially the control of the human environment for the purpose of creating human comfort and well-being. A natural cleavage will not be prevented by a "dog in the manger" attitude based on historical priority or superficial knowledge.

It is to be hoped that American engineers will defend and preserve their heritage of participation in public health activities by continuing to acquire the fundamental, non-engineering knowledge necessary for the comprehensive pursuit of their sanitary work; furthermore, that by participation in research they will continue to play a significant part in the advancement of this fundamental knowledge in its relations to engineering practice. It is to be hoped, finally, that the diverse groups of engineers engaged in the various branches of sanitation will be bound together as sanitary engineers by a unifying spirit of public service and a common interest and expertness in the protection and promotion of the public health.

H. G. Dyktor,[23] Assoc. M. Am. Soc. C. E. (by letter).[23a]—Mr. Bloomfield has ably covered the subject of the problem of industrial sanitation. There is one point, however, that should be stressed particularly, since the purpose of the Symposium seems to be to arouse the interest of all engineers in the necessity for industrial sanitation—namely, the extent to which engineers in industry can cooperate with the industrial hygienist without requiring the specialized knowledge and experience of the latter. This cooperative effort should be motivated by the desire to remove hazards to the health of the plant worker. Industry realizes that it is good business to have healthy workers, and engineers should realize it also. Moreover, only common sense and its intelligent application are necessary to effectuate this cooperation.

The industrial hygienist requires cooperation from plant officials on the following three items which are basic to all plants. This does not mean that they are the only ones but it does mean that they are the simplest, least expensive, and of general and immediate benefit both to the plant and its workers: (*a*) Housekeeping, (*b*) general sanitation, and (*c*) maintenance of protective equipment.

(*a*) *Housekeeping.*—By "housekeeping" is meant the systematic arrangement of the equipment, materials, and products in a plant. Under this heading the following items are to be found: Cleanliness, orderliness, and such regular routines as wet or vacuum sweeping of floors and joists, cleaning windows and electric light bulbs, etc. It is logical to insist on a clean and orderly plant because such a condition will be conducive both to safety and to a sanitary environment for the workers. It is also the base line from which further and more involved improvements may originate.

(*b*) *General Sanitation.*—The provision of a safe water supply, proper sewage disposal, and the absence of any cross-connection between these two

[23] Chf. Industrial Hygiene Engr., State Dept. of Health, Detroit, Mich.

[23a] Received by the Secretary January 17, 1940.

are included under "General sanitation." It includes also the provision of sanitary and approved drinking water dispensers (no common cups or glasses), individual towels (either paper or cloth, but not common towels), satisfactory and adequate toilets and urinals (properly separated for sex) etc. Welfare rooms, such as locker rooms, wash rooms, etc., should be light and clean. Illumination and ventilation of the plant should also be satisfactory and at least should meet minimum requirements.

(*c*) *Maintenance of Protective Equipment.*—Mechanical and protective equipment is expensive and should be protected by proper maintenance. Moreover, its efficiency depends on such maintenance. Sporadic and intermittent efforts in that direction are not as desirable as a carefully planned and regularly executed procedure. For instance, battered hoods should be reshaped to their original form, ducts should be examined for clogging, broken seams and other openings, all of which interfere with, and reduce the efficiency of exhaust equipment. Likewise collectors must be emptied and cleaned at regular intervals. Personal equipment such as respirators should have their filters, etc., replaced when filled, so as to reduce unnecessary resistance to breathing, and holders should be cleaned and kept in dust-free places. Positive pressure helmets should have their reducing valves checked; compressed air should be filtered to remove oil and should be analyzed periodically for carbon monoxide. Many other precautionary measures could be listed.

The foregoing brief discussion contains no sanitary measures that could not be accomplished by the average engineer and yet, if these items were attended to a great load would be removed from the shoulders of the industrial hygienist who could then spend the time thus saved on other matters requiring his specialized knowledge and experience.

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DISCUSSIONS

BRIDGE AND TUNNEL APPROACHES

Discussion

BY MESSRS. DEAN G. EDWARDS, AND JOHN W. BERETTA

DEAN G. EDWARDS,[8] M. AM. SOC. C. E. (by letter).[8a]—It is probably true that between 25% and 50% of the total cost of modern vehicular bridges and tunnels is expended on their approaches, but unfortunately this has not always been the case, particularly in times long past. The present inefficient use of some of the older bridges over the East River in New York City is evidence that too little consideration was given to approaches when these bridges were built; or it may be fairer to state that traffic has changed so radically that such approaches as were provided have become obsolete.

Brooklyn Bridge is used for only a fraction of its potential capacity because traffic cannot leave or enter the Manhattan end except by two traffic lanes in each direction, and it is subject to repeated interruptions of serious proportions because of pedestrians on Park Row and because of the light-control system. A costly major operation must now be made if the bridge is to be used to its potential capacity.

The identical conditions do not exist for all the older bridges. The Manhattan and Williamsburgh bridges are more efficient in traffic movement than the Brooklyn Bridge because the approach streets are either wider or more numerous, and there is some opportunity to even the "humps and hollows" of the traffic curve which are caused by the traffic lights.

Among the more troublesome traffic points are the approaches of the Queensborough Bridge. This bridge has five traffic lanes on the main level and two lanes on an upper level. The direction of traffic on the upper level conforms to the direction of the heavier load. It is Manhattan-bound from midnight to noon and Queens-bound from noon to midnight. The upper road was first used in 1931, 22 years after the bridge was opened, and a diversion viaduct was built in Queens so that the upper roadway traffic enters and leaves some blocks distant from the main-level traffic. In Manhattan, the upper

NOTE.—This paper by John F. Curtin, Jun. Am. Soc. C. E., was published in November, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[8] Cons. Engr., Borough of Manhattan, New York, N. Y.

[8a] Received by the Secretary January 10, 1940.

roadway turns to the south over 59th Street and exits into either 58th Street or 57th Street through a right of way a little east of Second Avenue.

The main level of the bridge opens into Second Avenue, a light-controlled thoroughfare with frequent interruptions of traffic resulting from the 90-sec light cycle. An exit into 60th Street and an entrance from 59th Street help a little toward the development of bridge capacity, but traffic jams are frequent at peak hours. The lack of proper approaches has not only diminished the traffic flow over the bridge but has had an extremely adverse effect upon the street traffic and property values in the area in proximity to the bridge. Second Avenue is usually closed to north-bound traffic at the bridge (except buses), and instead of the bridge traffic making up the difference between Second Avenue capacity and the normal north-bound traffic, the normal north-bound traffic on Second Avenue is forced to detour. The traffic load of First Avenue, the nearest wide thoroughfare without elevated railroad columns, is about at its ultimate capacity during peak hours, and Second Avenue fails to help by performing its rightful function. The nearby crosstown streets of this vicinity are often filled to capacity, and a blight on property has resulted.

Plans have been devised for easing the present undesirable situation. Two fundamentally different schemes have been proposed. One provides for depressing the bridge traffic lanes under Second Avenue into a plaza which would occupy the full width of the block between 59th and 60th Streets, nearly to Third Avenue. A new street cut through to 57th Street will permit the traffic to disperse through the cross streets. A study of this plan reveals that the main points in its favor are that it returns Second Avenue to normal use as a north-south artery and that a broad plaza is provided as a reservoir for equalizing the interruptions of flow caused by the stop lights. The objections to this plan are: (1) Its high cost; (2) the fact that much of the traffic over the bridge will desire to reach the East River Drive, and this routing would bring it to the wrong side of Second Avenue; and (3) the moving of the bridge outlet to the west side of Second Avenue would not in itself solve the problem at all, but would merely move the congestion a half a block toward the west.

The alternate solution entails a widening of Second Avenue from 100 ft (between building lines) to 175 ft for about seven blocks, and passing the normal traffic of Second Avenue by the bridge entrance in a four-lane tunnel with staggered entrances and exits. The widened avenue would provide enough lanes for reservoir space for equalizing the stops caused by the light system, and the bridge traffic, at full capacity, would not exceed the capability of the approach streets to absorb it. The elevated railroad on Second Avenue complicates the problem, but the proposed early demolition of this structure may remove this obstacle. In any case, this problem must be considered in all its aspects, including the entrances and exits to both the north-bound and south-bound roadways of the East River Drive.

The East River Drive from 93d Street to 125th Street was constructed as an approach to the Triborough Bridge. It will become a part of an express highway without light control, having three lanes of traffic (11 ft per lane) in each direction, and a limited number of entrances and exits.

The importance of estimating bridge capacity in terms of the capacity of approach streets to absorb the traffic cannot be overemphasized.

There is considerable doubt as to the usefulness of traffic circles as regulators of traffic. In outlying districts they seem to be fairly satisfactory, but they are a nuisance to through traffic which must slow down to a greatly reduced speed. The objection to them in Manhattan is not that they are uneconomical because of the high cost of real estate but that they are unsatisfactory under heavy traffic conditions. Traffic circles, under heavy traffic, induce many minor accidents, slow traffic unreasonably, are difficult to operate, and do not obviate the use of lights. The experience in Washington, D. C., where Massachusetts Avenue is depressed under Thomas Circle, is rather conclusive. Any one who has often driven a car around Du Pont Circle during peak load hours cannot retain enthusiasm for traffic circles in large cities.

The examples of present problems created in Manhattan because of failure to foresee the vast volume of traffic, which has developed, and to plan accordingly are cited to show the necessity for the substantial part of bridge costs which Mr. Curtin mentions as properly expended on approaches. Earlier realization of this need and proper planning for adequate approaches would have simplified the present problem of reconstruction immensely. Even the admirably designed approach system of the Lincoln Tunnel has created traffic problems in Manhattan, because of bus concentration outside the limits of Dyer Avenue. It is inevitable, perhaps, that capacity traffic of tunnels and bridges will always create traffic problems in densely populated areas outside the distribution plazas.

JOHN W. BERETTA,[9] ESQ. (by letter).[9a]—Results of careful study and research are evidenced throughout Mr. Curtin's comprehensive paper. He is to be congratulated upon a concise and compact treatment of a subject which, because of its diversity and many possible ramifications, might have extended to an almost impossible length. He has indeed set a good example by controlling his subject in as efficient a manner as he suggests for the disposal and expedition of traffic in the approaches to bridges and tunnels.

One quite logical conclusion that may be drawn from this paper is the fact that only quite large and monumental bridges and tunnel developments have received adequate and serious design effort on approaches. In other words, the amount of approach planning for a particular structure is roughly proportional to the cost of the main structure, its size, and its importance as a link in the highway system it serves. There is no doubt that even the smaller and less important crossings could be improved materially if more attention were given to approach design study, but in the past this has been consistently overlooked or purposely disregarded due to limited financing.

Another significant fact emphasized in this paper is that all of the structures discussed as illustrative examples are of rather recent construction. This would lead to either of two natural conclusions: First, that older structures of comparable magnitude and importance did not have comparable approach problems because of less density and velocity of traffic; or, second, that the importance of approach problems was overlooked, either purposely or because a

[9] Cons. Engr., San Antonio, Tex., and President of the Am. Toll Bridge Assoc.

[9a] Received by the Secretary January 15, 1940.

lack of study and knowledge prevented recognition of their importance. A study of some crossings older than those discussed by Mr. Curtin undoubtedly will reveal that both conclusions may be valid although they may vary in individual cases. This may serve to some extent to explain his statement that "relatively little of the diligent research and analysis * * * has been made available to the engineering profession."

The statement regarding the paramount importance of the kinetics of vehicles is very significant in that it tends to bring to mind the fact that older structures did not have comparable approach problems. Also it tends to prove that bridge and tunnel approaches are highly susceptible to obsolescence due to changes in type, density, and velocity of traffic which, in many cases, can become serious before the obsolescence of the structure itself, regardless of the cause. There are many structures in use today which, even if they were actually built for traffic of "horse and buggy" velocity, could accommodate modern high-speed traffic if their approaches were amplified and modernized properly. Their usefulness, based on Mr. Curtin's data, could be increased from 50% to 100%, and such reconstruction might avert the necessity of constructing an entirely new facility.

As surely as adequate foundations are necessary for the stability, load capacity, life, and safety of any structure, so are adequate approaches necessary for the utility, life, economic value, traffic capacity, convenience, and traffic safety of a structure. This analogy is the more significant when one ascertains that the percentage of cost of the substructures of a crossing are more than likely to be about the same as the percentage of cost of the approaches. It has been difficult, in the past, to convince laymen and even some engineers of these facts, but the evident progress, as outlined by Mr. Curtin, is most encouraging and certainly is a trend in the right direction.

During the past few years the toll bridge industry in general has been under rather withering fire from governmental highway agencies seeking complete elimination of the toll bridge as a public servant, so that the traveling public can be provided with "free" bridges. In this campaign to provide these "free" bridges (so-called because no direct toll is paid although indirectly a toll is exacted of every taxpayer whether or not he uses the bridge), arguments in favor of the "free" bridges have been offered which have been confusing and misleading to the general traveling public. Because of their approaches, which due to the rapid change of traffic volume and speed are becoming inadequate, many crossings adequate in all other respects have been condemned unjustly and a loud clamor has been raised for superseding or duplicating facilities when only approach improvements and extensions are needed for the existing facility.

To the toll bridge industry, therefore, the general problems of bridge approaches are increasingly important. Not only must new structures be provided with properly designed approaches, but old structures, if they are to survive these—for the most part unwarranted—attacks, must be rehabilitated through rational modernization and amplification of approaches. Toll bridges have been hailed through the ages by economists as the fairest and most equitable crossings because they are paid for by the actual users of the facility and not by long-suffering taxpayers who may never even see them.

Despite the fact that it is not generally known, it is true that the toll bridge, as well as the toll road, type of facility is again growing in use and acceptance by the public. The toll bridge industry originally was founded only on the basis of the investment of private capital. It is only in comparatively recent years that public and quasi-public agencies have constructed toll facilities financed through the issuance of self-liquidating securities. The privately financed toll bridges, therefore, are pioneering ventures and as such are entitled to recognition for the performance of a public service in a time of necessity. It is significant that in Mr. Curtin's paper most of the specific examples are crossings of the toll type, paid for by revenues instead of taxes.

Of the various groups and types of existing bridges which need modernization of approaches, none presents more acute problems than international structures, particularly those along the Mexican border. Here, however, the complicated problems of traffic capacity and distribution come at the exit instead of at the entrance to the crossing. Each international crossing is provided with inspection facilities for four governmental agencies—namely, the Public Health Service, the Bureau of Customs, the Department of Agriculture, and the Immigration Department. These facilities are in duplicate, with the American and equivalent foreign agencies each occupying one approach to the structure. The time element involved in the inspections by these agencies enormously complicates the handling of traffic, especially when it is necessary to revise tourist baggage and conduct the other business of a port of entry.

The most important and pressing problems developed by Mr. Curtin are those of entrance; and his exit problems are chiefly questions of geographical distribution and the clearing of off-bound vehicles through cross traffic. In connection with an international crossing the problem somewhat reverses and the chief concern is one of exit. In these cases, however, off-bound traffic must be sorted into three major groups—local, freight, and tourist, with each group requiring a distinct handling. The consideration of geographical distribution and cross traffic clearance at the exit is of minor importance when compared to the other problems.

In the days before the introduction of through trunk highway systems all of the traffic on these international bridges was purely local in character with little complication or delay involved in governmental inspections. At that time all freight and tourists with baggage were carried on the railroads. Now, however, the situation is changing to completely outmode the existing approaches, which, with few exceptions, are of the "direct street or highway extension" type as described by Mr. Curtin. With privately-owned tourist automobiles traveling far afield and with an enormous tonnage of freight being carried by motor trucks, acute conditions are arising from the long queues of off-bound vehicles extending back on the structure proper while awaiting the unwaveringly complete inspections. On-bound traffic at each end of the structure can be speeded through utilization of the known methods suggested by Mr. Curtin, but the expedition of off-bound traffic is a problem which is still under process of practical solution. Probably this solution will be found in the construction

of reservoirs where cars detained for inspection may be parked out of the way of local traffic not requiring luggage revision.

The first tangible recognition of the necessity for approach improvements to an international crossing has come from the United States Congress. In 1939 this body appropriated $290,000 for the construction of a Port Terminal Building at the existing privately owned and operated bridge at Laredo, Tex. This improvement will be of great benefit in accommodating the increasing port business at this point, which is the greatest of any Mexican border port. Experiences gained in this development by the Federal Works Agency will be highly valuable when consideration of other similar problems is undertaken.

The International Highway Bridge between Laredo, Tex., and Nuevo Laredo, Mexico, is a large mounumental concrete arch structure which forms an important link in the Inter-American Highway extending from Canada almost to Central America and which, ultimately, will extend to South America. With the exception of the exit facilities (outmoded because of a change in the type of traffic handled), the bridge itself is considered adequate for all traffic requirements. With the completion of the new governmental port terminal facilities, which will modernize the approach completely, the bridge again can function properly to accommodate the full capacity of its roadway. This is a clear-cut example of a toll facility that can be restored to full usefulness by approach amplification without the need of an unnecessary duplication of the facility.

At first the problem at Laredo is one of segregation or separation of various types of traffic at the exit, because each required different types of inspection. The largest volume consists of local passenger-car and empty-truck traffic whch normally can flow with only casual inspection and a slight delay equivalent to that necessary for the payment of toll at the entrance. The next to merit consideration is the very troublesome loaded-truck traffic which is subject to inspection delays of from a few minutes to several days in extreme cases. The most serious and troublesome problem is experienced with passenger cars laden with tourists, varied baggage, and huge quantities of merchandise souvenirs, the problem being complicated further when the tourists are aliens. Added to this conglomeration are the additional problems of loading and unloading platforms, office facilities, rest rooms, waiting rooms, detention spaces, fumigation and disinfecting facilities, warehouse storage, and many other related factors.

In the solution of the problem many ideas are being borrowed from railroad practice which long has been concerned with segregation and rational handling of varied types of traffic on a restricted basis of entrance and exit. Also ideas have been considered from the practice of water-borne traffic handling at ports of entry along the seacoast. Together with these, however, ideas are also adapted from the latest bridge approach practice as described by Mr. Curtin, and partake of the essential features of nearly all of the types he describes. The local traffic travels in the same manner as it would on a direct street or highway extention with some tapered plaza and feeder connection modification. The other types of traffic must be accommodated by a series of segregating reservoirs designed to hold a maximum of each type. The whole must be

correlated with the other features of a port-of-entry terminal as described previously.

It can be seen, therefore, that the bridge-approach problems of an international crossing are much more complicated than even the larger interstate or intrastate crossings. With the rapid increase of international highway traffic, particularly to Latin America, the problem is really an acute one, and a satisfactory solution must be found if this traffic is to be handled adequately. The studies at Laredo undoubtedly will afford a satisfactory solution of that particular problem and will point the way to the final answer to the special problems of other international crossing locations, either Canadian or Latin American.

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DISCUSSIONS

ANALYSIS OF LEGAL CONCEPTS OF SUBFLOW AND PERCOLATING WATERS

Discussion

By Donald M. Baker, M. Am. Soc. C. E.

Donald M. Baker,[25] M. Am. Soc. C. E. (by letter).[25a]—A number of inconsistencies and fallacies exist in many court decisions relative to ground-water occurrence and behavior. The authors are to be congratulated in bringing these fallacies into clear focus and it is hoped that the influence of the paper will be reflected in future decisions dealing with this topic.

Much of the early investigational work dealing with ground-water hydrology was occasioned by a series of court cases which arose out of the 9-yr dry cycle that occurred in Southern California around the turn of the century, and many of the legal principles laid down and classifications adopted by the courts in the western states had their beginnings in these cases.

The two decades 1920 to 1940 have seen a great accumulation of factual information upon this topic, a material increase in the knowledge of the causes and effects of the occurrence and behavior of ground water, as well as great improvement in the technique of collecting basic data relative to ground water. Blame for the failure of the courts to recognize many of these developments may lie partly with the technical experts who hesitate (because of a fear that such presentation will not "get over" with the court) to present ground-water situations in the light of the latest knowledge available regarding them, and partly with the courts themselves, who, for their knowledge of a very complicated technical subject, rely too greatly upon legal decisions written by men who, though well grounded in the law, are not familiar with the fundamental principles of ground-water hydrology. Thus a vicious circle exists, which it is hoped this paper may help to disrupt.

As stated by the authors, all ground water—that is, water occurring within completely saturated portions of the lithosphere—is in movement from the point where it enters the saturated zone to the point where it is discharged from it. If three axes, X, Y, and Z, each at right angles to the other, are taken

Note.—This paper by C. F. Tolman and Amy C. Stipp was published in December, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[25] Cons. Engr., Los Angeles, Calif.

[25a] Received by the Secretary December 27, 1939.

at any point in a body of ground water, with the Z-axis being assumed to lie in the direction of movement of the ground water, the Y-axis being vertical, and the X-axis horizontal, four classifications of ground water may be made, based upon the location of the formations which restrict movement along such axes, in which:

1. The principal restriction is against movement in a downward or $-Y$ direction, such restrictions as exist against movement in a $+X$ or $-X$ direction being located a considerable distance apart;
2. Restriction is against movement in a downward direction, along the $-Y$-axis, and also against lateral movement in a $+X$ or $-X$ direction;
3. Restriction is against movement upward and downward, in a $+Y$ or $-Y$ direction, such restrictions as exist against movement in a lateral direction along the $+X$ or $-X$ direction being located a considerable distance apart; and
4. Restriction against movement is in an upward and downward direction along the $+Y$ and $-Y$ axes, and also against lateral movement in a $+X$ and $-X$ direction.

In Class 1 would fall all free ground-water bodies, commonly known as ground-water basins or reservoirs, underlain by impervious or nearly impervious formations with the upper surface of the ground-water body free to move in a vertical direction with changes in volume of the ground-water body, movement of ground water being in the direction of the slope of this upper surface or water table. Although in most instances such bodies of ground water are confined laterally, the confining formations are located far apart. A profile of the water table of such body along the X-axis would be concave or convex, or even sinuous.

Class 2 would embrace subsurface water courses, the formations restricting lateral movement being located relatively close together, in terms of the length of the body along the Z-axis; and a transverse profile of the water table, although it might be slightly concave or convex, would more closely approach a straight line.

Ground-water bodies in Class 3 would be similar in shape to those in Class 1, except that all ground water in the body would be under pressure, whereas those in Class 4 would be similar in shape to those in Class 2, but would be under pressure, the flow of ground water in Class 3 and Class 4 being in the direction of the pressure gradient.

Distinctions between ground-water bodies in the four classes are not always clearly defined, and sometimes become a matter largely of degree, particularly in regard to restrictions against lateral movement. A body having lateral restrictions 200 ft apart might be placed in Class 2, whereas if similar restrictions were a mile apart it might be placed in Class 1. Bodies in Class 1 and Class 3 have general characteristics more or less similar to surface lakes, whereas those in Class 2 and Class 4 have characteristics more akin to those of surface streams.

Many ground-water bodies may fall in one class in one portion, and another class in another. For example, ground water in a large valley may fall in Class 1 in the upper section or along the sides, and in Class 3 in the center or at

lower elevations. Furthermore, a large ground-water body may be separated into a number of subdivisions, either within the same or in different classes. A body in Class 1 may be divided into a series of basins, each with a water table at different levels, separated from one another by underground dikes, faults, or ridges over which the ground water cascades to lower levels. Reductions in the cross-sectional area normal to the direction of movement will cause like subdivisions in bodies in classes 2, 3, and 4.

Legal controversies over utilization of ground water, other than those dealing with the relative rights and priorities of users from a ground-water body, usually fall into two general classes:

(*a*) The supply for persons claiming superior rights to the use of such ground water is asserted to be reduced through the diversion by the person complained against; and

(*b*) The elevation of the water in the wells of persons claiming superior rights to the use of ground water is asserted to be lowered through the diversion by the party complained against.

Nearly all of the cases with which the writer is familiar are based upon claims made under Class (*a*), the plaintiff asserting that actual or threatened diversion by the defendant would diminish his supply of water, and the court making its ruling on this basis, when, as a matter of actual fact an ample supply existed to satisfy plaintiff's right, and the sole effect of defendant's diversion was to lower the elevation at which the water stood in plaintiff's well and require him to lower his pump.

There appears to be little or no discussion in ground water cases as to whether the owner of a superior right to use ground water also has the right to have the elevation of the water in his well maintained in the same manner as existed prior to diversions under inferior rights. Current knowledge and technique in ground-water hydrology allow the determination, with a reasonable degree of accuracy, of the quantity of ground water which may be diverted safely from a ground-water body over a given period of time without exceeding the supply of water which the body will receive during such time; and from such a determination the question of the probable diminution of supply necessary to satisfy superior rights may be settled.

Numerous situations exist in which increased draft upon a ground-water body will actually increase the quantity of water which may be diverted from it. Such increased draft during periods of subnormal supply enlarges the storage capacity of the ground-water body and allows more water to be impounded during periods of more than normal supply. The same situation exists when the storage capacity of a surface reservoir is increased—the increase in storage capacity allows an increase in the draft from the reservoir. Such increased draft upon ground-water bodies, however, usually results in a considerable lowering of the water level in wells used to divert from it, over extended periods of time.

If future decisions should hold that rights to divert and use water from ground-water bodies include the right to maintenance of the elevation of the water in the wells through which such water is diverted, it would be a severe blow to the interest of conservation and highest utilization of such supplies. There is a great need for clarification of this phase of ground-water law.